Quantum Properties of Black Holes:
Further Understanding the Double Cone Spacetime

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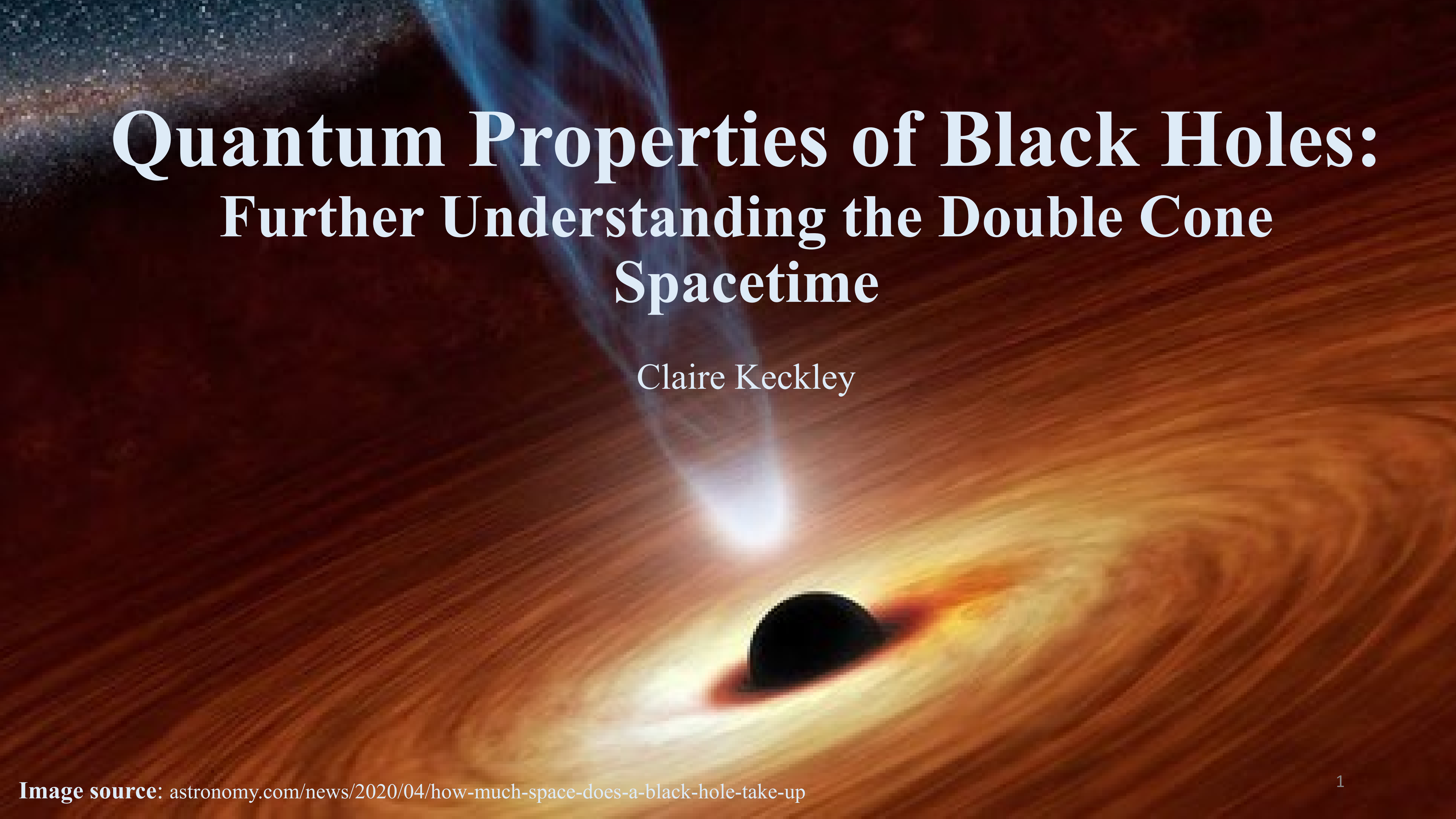
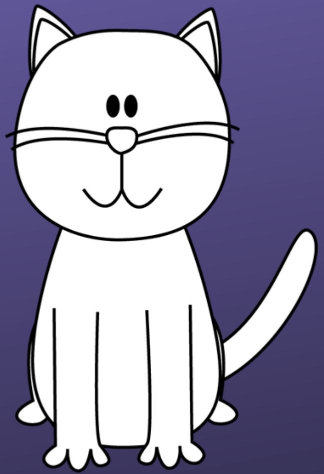


Image source: astronomy.com/news/2020/04/how-much-space-does-a-black-hole-take-up

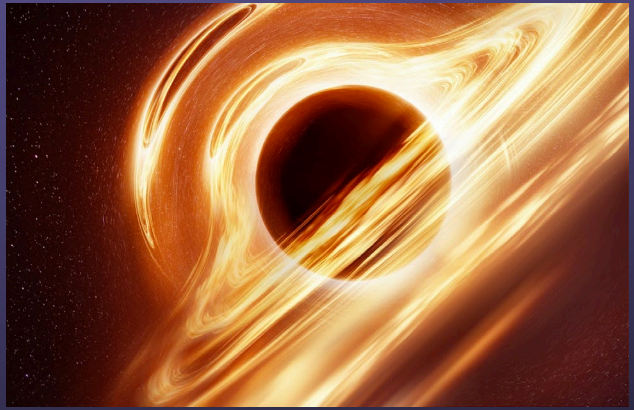
The fundamental problem:
Tension between quantum mechanics and relativity

Black hole information paradox

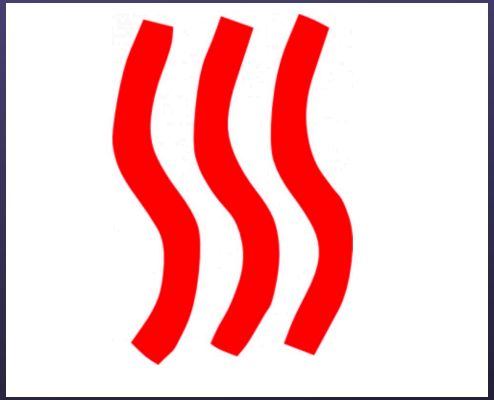
- According to Hawking, black holes evaporate into thermal radiation (heat).
- What happened to the information (physical properties) stored in the black hole?



Cat



Black hole



Heat?

Image sources: mycutegraphics.com/graphics/cats/black-white-cat.html; sciencefocus.com/news/researchers-verify-extremely-odd-black-hole-physics/; clker.com/clipart-heat-symbol-1.html

Framework of one solution (Part 1)

1. Steve Shenker and Douglass Stanford at Stanford University wanted to help resolve this tension by showing that black holes do behave as quantum systems.

    - Quantum systems, such as atoms, do not destroy information.

2. One way to support this idea is to show that black holes have discrete energy levels.

3. Rather than computing each energy level, they found it easier to compute a certain sum over the energy levels.

Framework of one solution (Part 2)

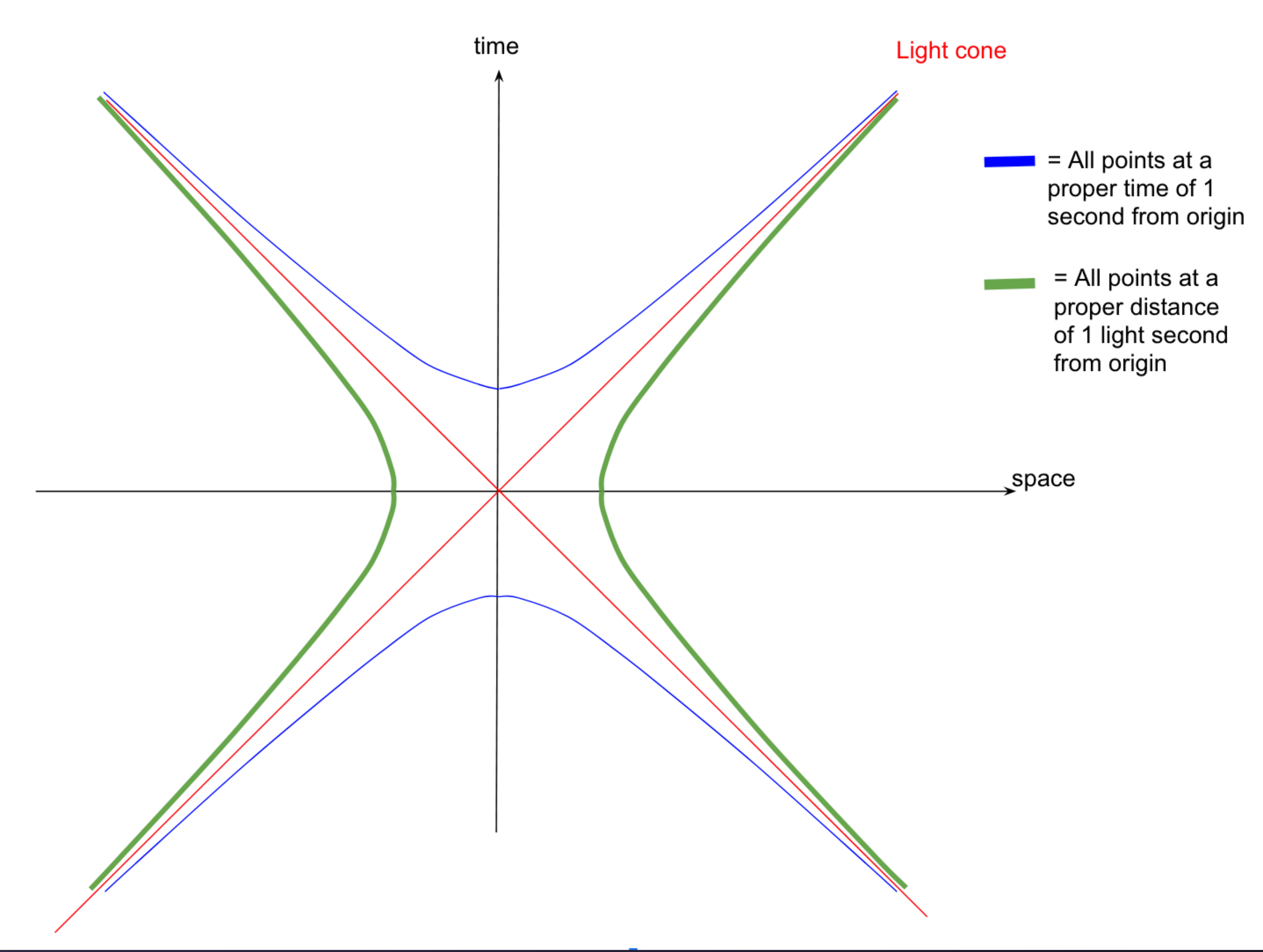
4. This sum can be computed by a path integral that sums over spacetimes having two boundaries and where time is periodic in each boundary.

5. This path integral can be approximated by $\boldsymbol{e^{iS_0/\hbar}}$, where S_0 is a function of the spacetime geometry.

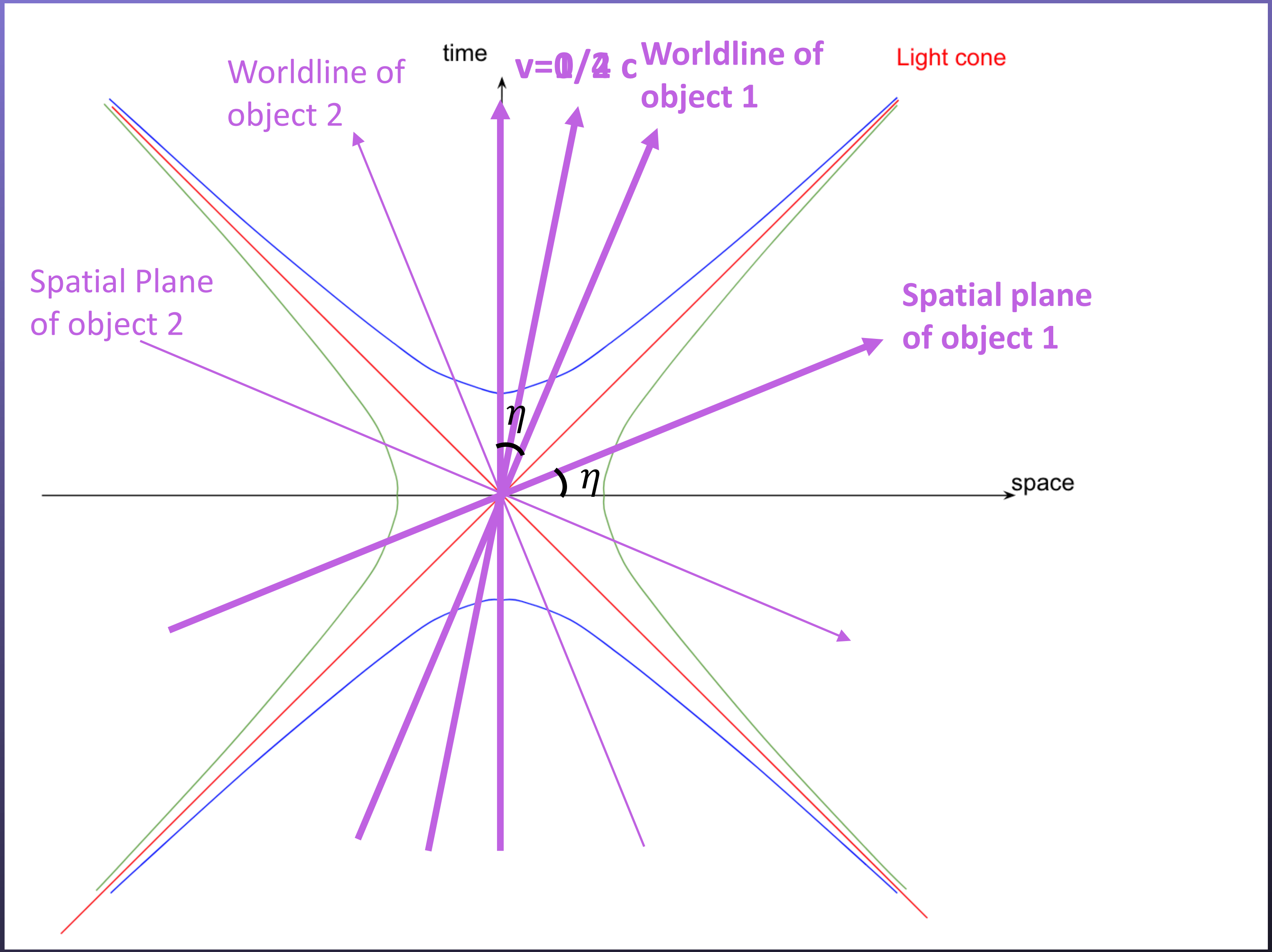
6. This function can be found using classical solutions of Einstein's equations that satisfy the above boundary conditions.

<u>Summary:</u> According to researchers at Stanford, we can use classical solutions of Einstein's equations to show that black holes behave as quantum systems, which will ultimately help us understand how quantum mechanics fits with relativity.

Minkowski Space-time Diagram

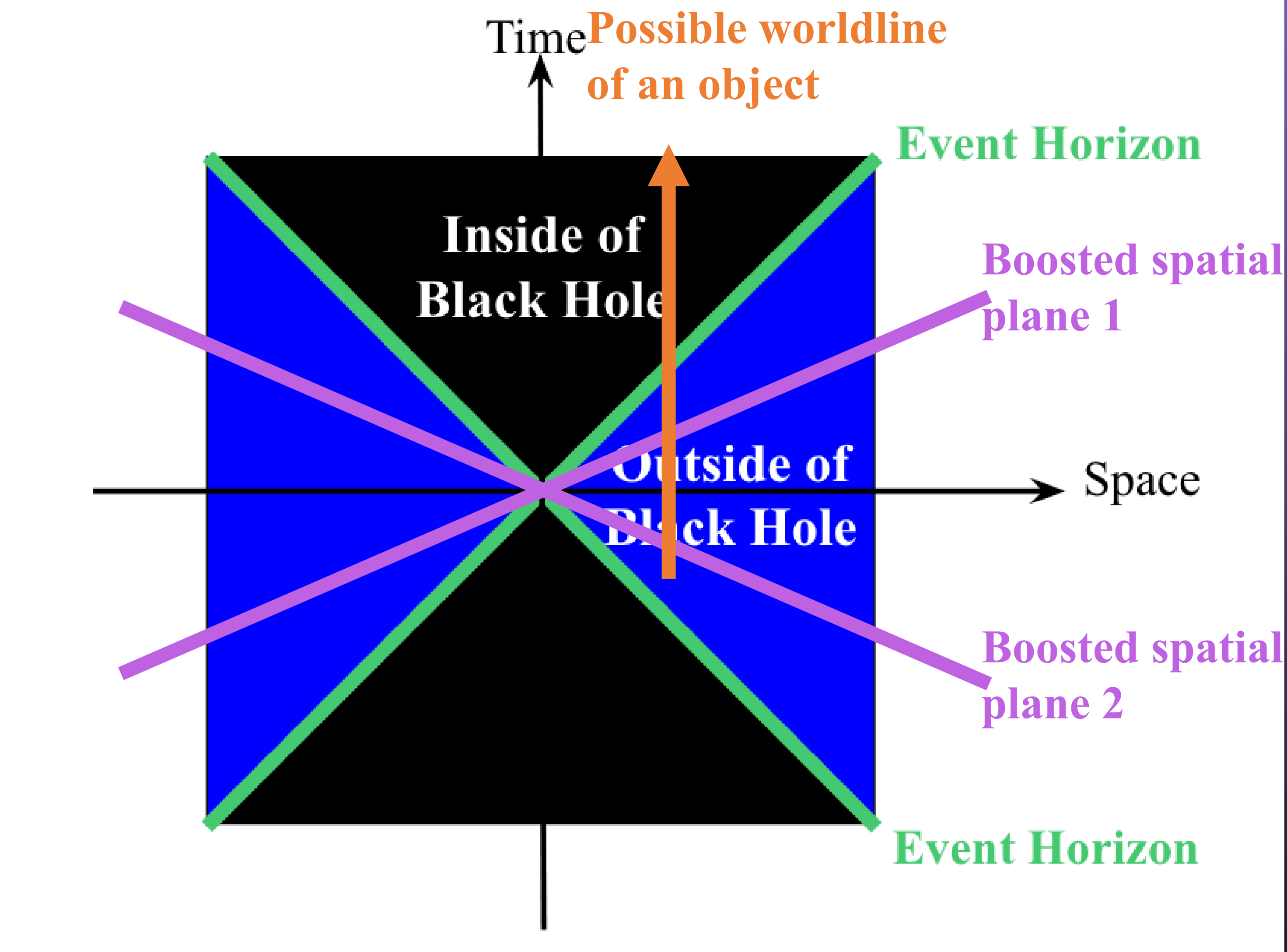


Boost Parameter



What is a black hole?

Black hole diagram



Making a new topology: Quotient the two planes of space

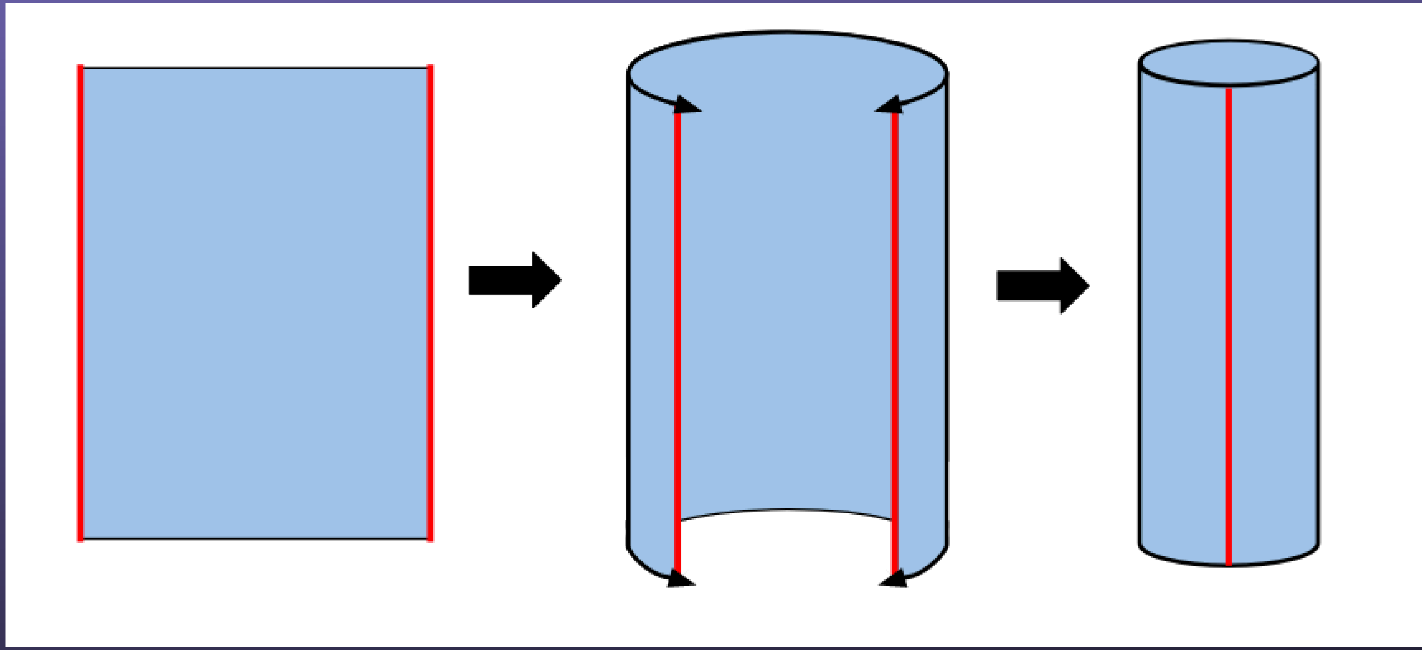
To form the black hole topology that the researchers at Stanford created, we must take the quotient of the blue region by identifying the two planes of space.

Identifying the planes essentially means “gluing” the two together to form a cone.

Each point of space outside of the planes (and within the same quadrant) is mapped to a point on the cone.

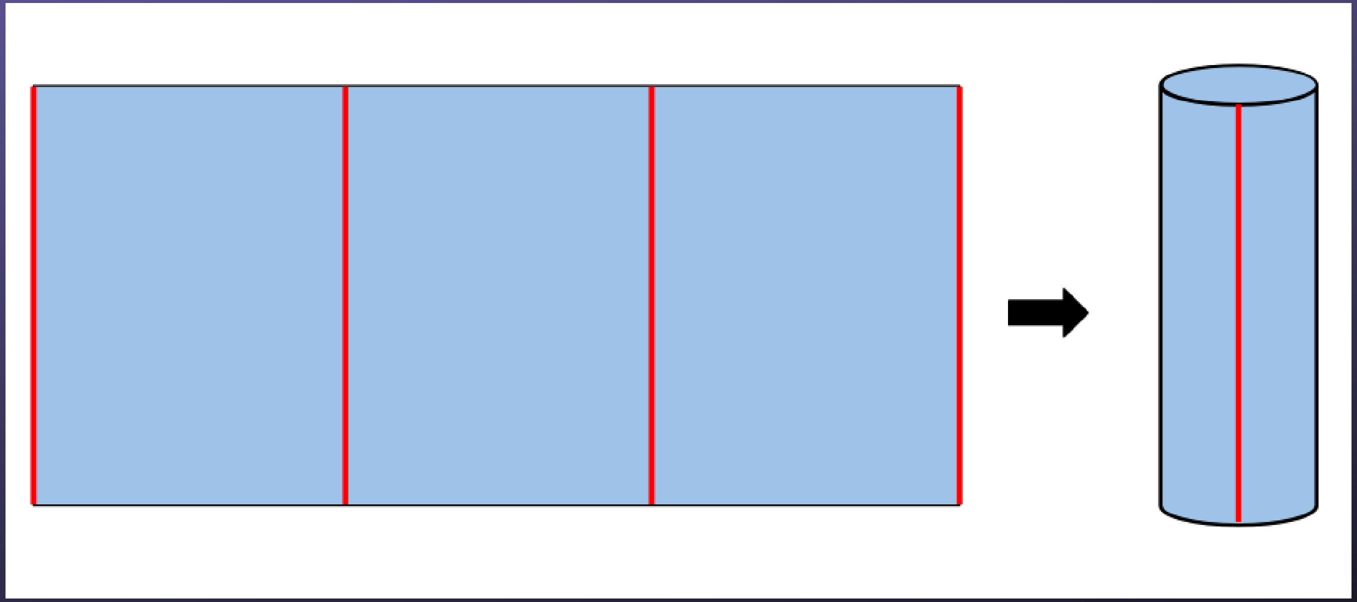
Quotient example (Part 1)

Imagine gluing two ends of a rectangle together to form a cylinder. This process is called identifying the lines.



Quotient example (Part 2)

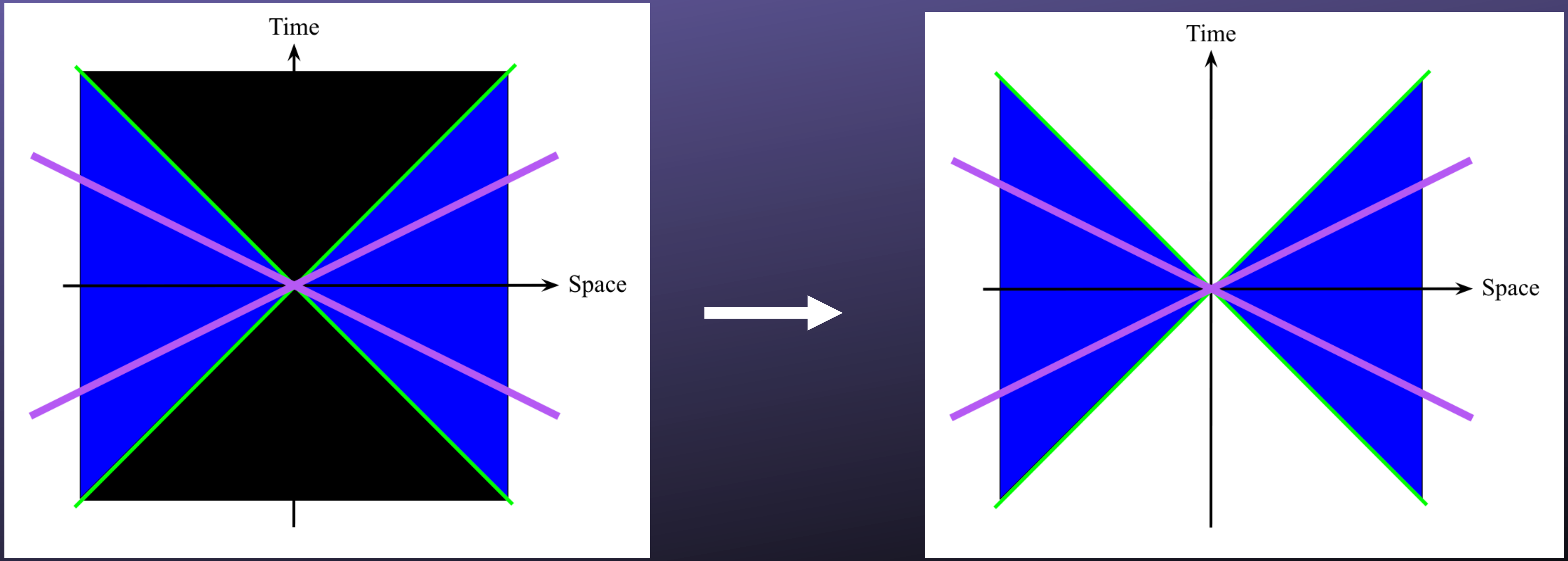
- We can also imagine taking a series of rectangles and folding them into a cylinder of the same circumference as our single-rectangle cylinder.
- Each subsequent rectangle will create a cylinder which overlaps on top of the original cylinder.
- We say that the points on these rectangles are “mapped” to points which they lay on top of in the original rectangle (and, subsequently, the original cylinder).
- This cylinder is called the quotient.



Step 1 of taking the quotient

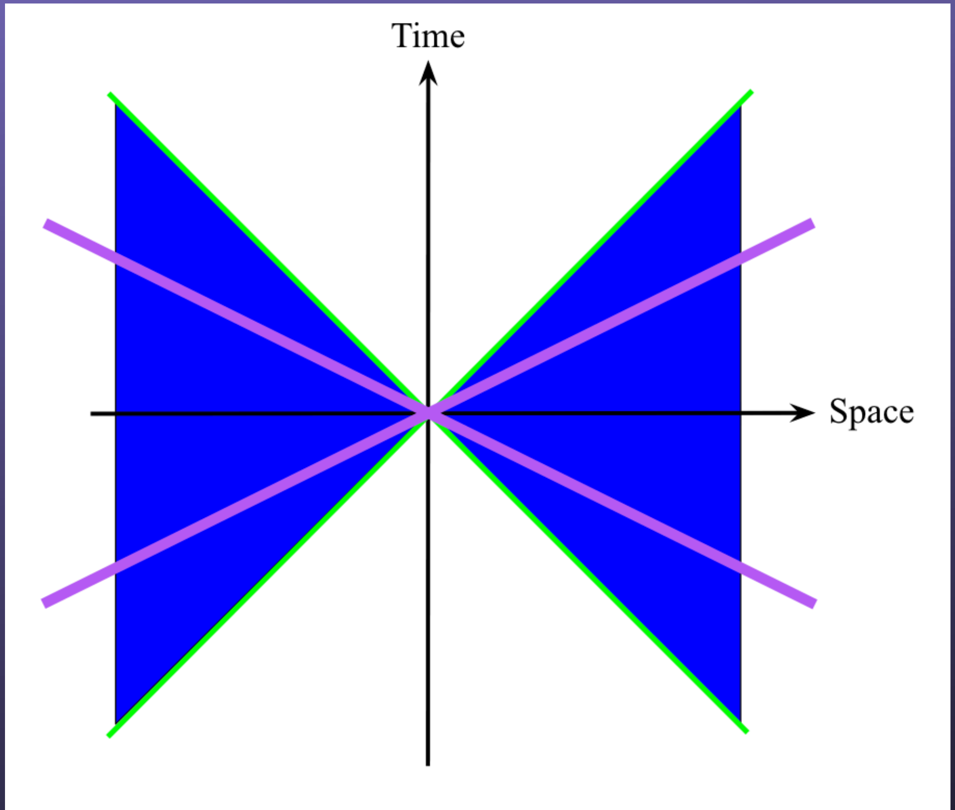
Every point in the region which we quotient must be able to be mapped to the region between the planes of space by the boost parameter.

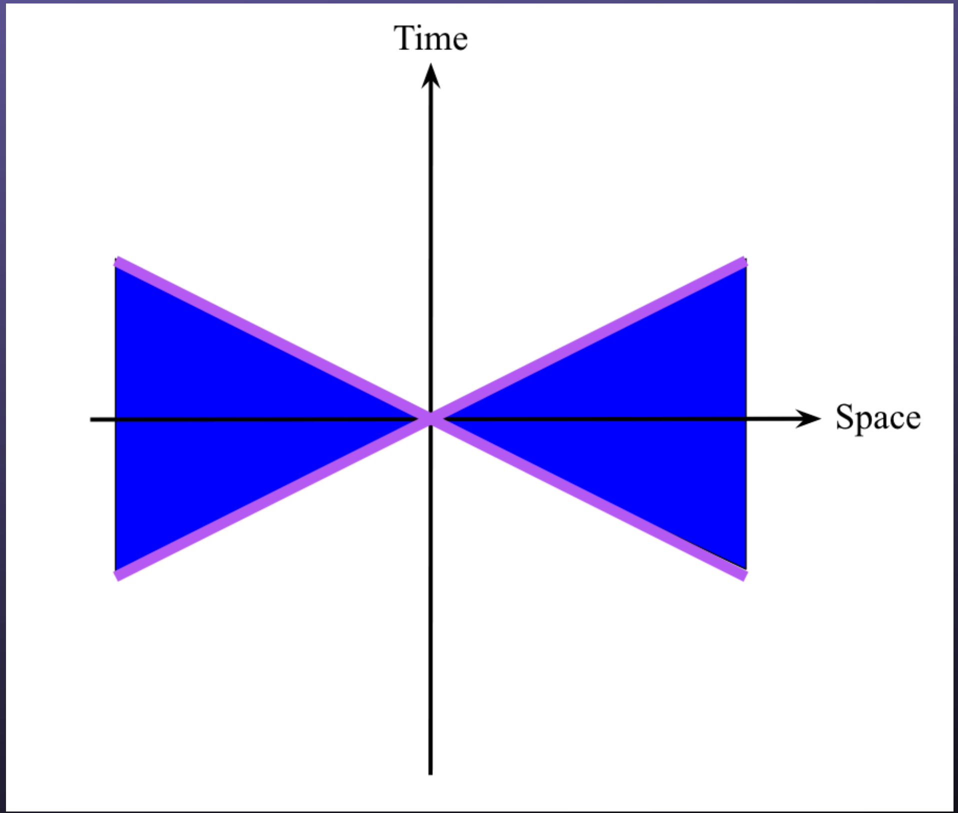
Therefore, we will take out the top and bottom quadrants.



Step 2 of taking the quotient

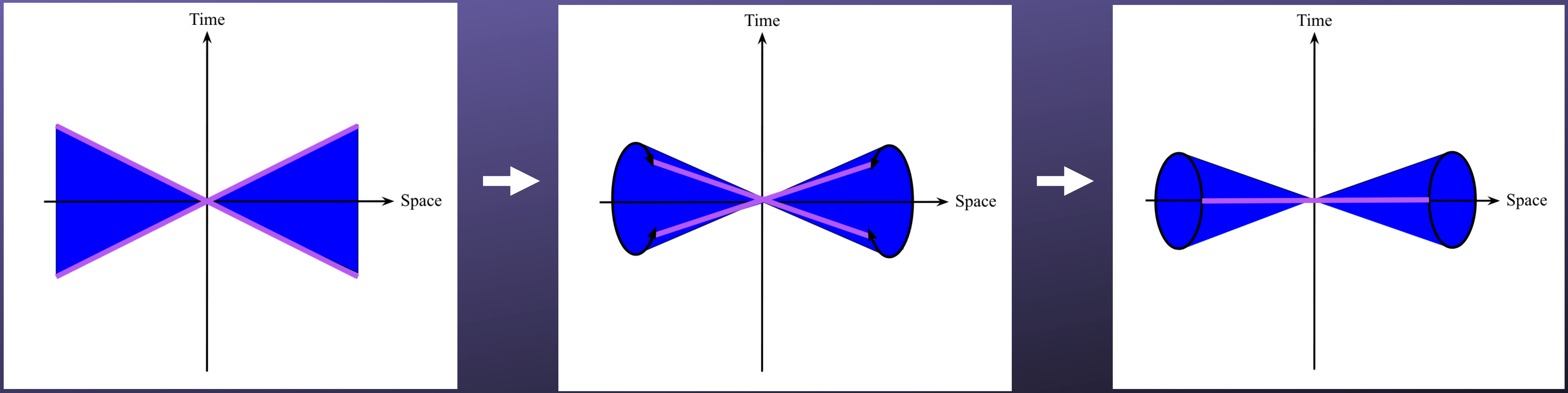
Next, we will take all of the points outside of the two planes of space and map them to points within the region.



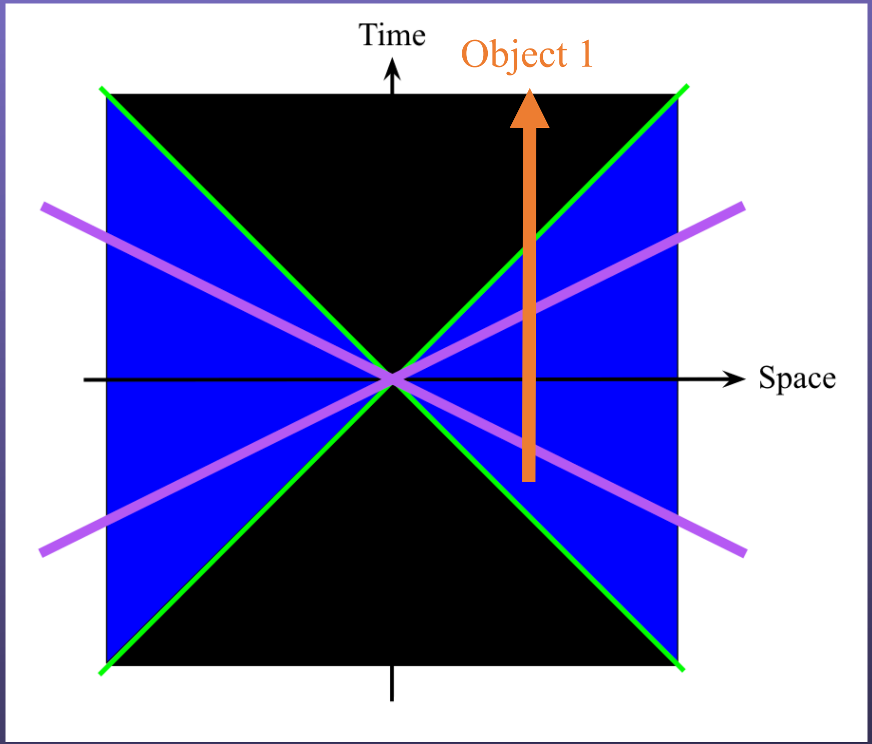


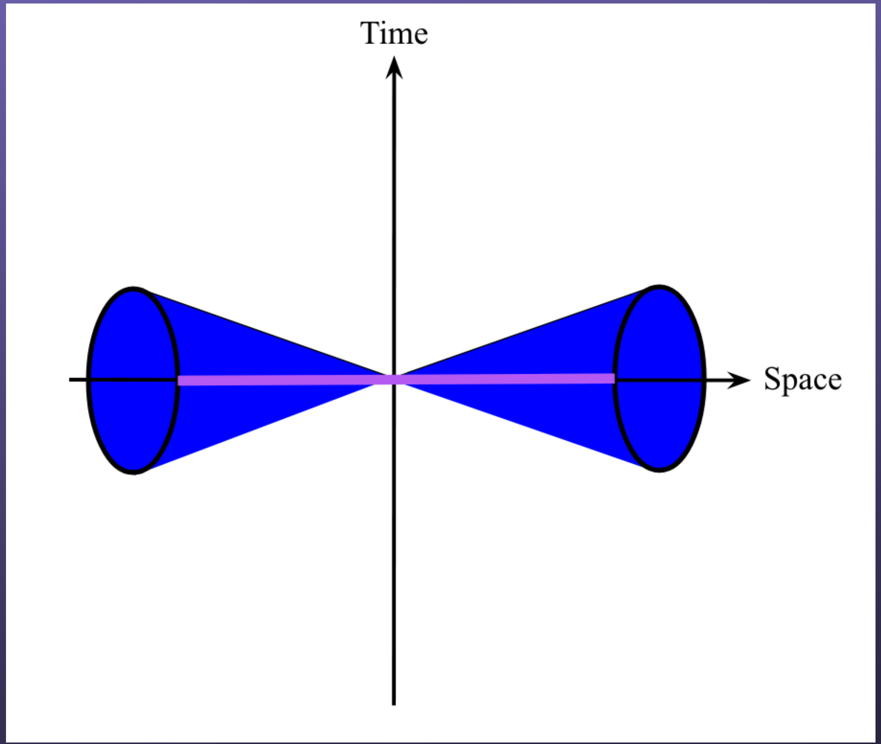
Last step of taking the quotient

Finally, we roll this region into the cone by identifying the two planes of space. This creates what is known as the double cone spacetime.



Each point within the blue region can be mapped to a point on the cone. However, an object in freefall that starts outside of the black hole in the flat space will eventually reach the black hole region, which cannot be mapped to any points on the cone.





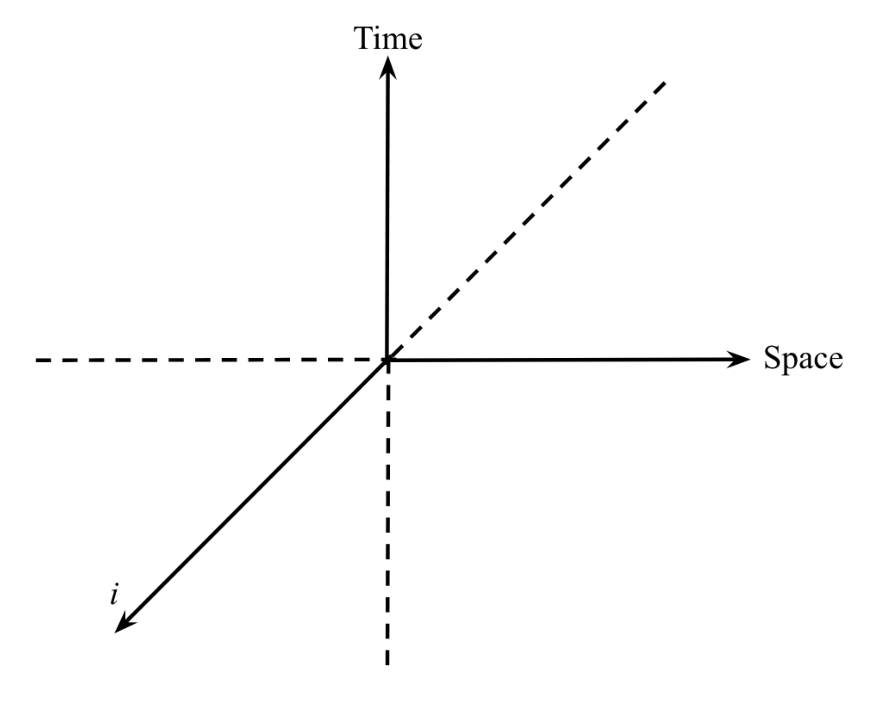
The questions:

<u>What happens to this object in the double cone spacetime?</u>

<u>Do we need to add back in the black quadrants to get a complete picture?</u>

Method of finding the solution

Shift space and time slightly into the complex plane before taking the quotient.



Note that space and time will both have an imaginary dimension after the shift, so the graph will actually become 4D. The above picture shows a particular part of the graph where one of the imaginary components is $0i$.

Why can we do this?

As mentioned earlier, the calculation of the black hole's energy levels will involve an integral over the spacetime geometry.

According to the Cauchy integral theorem, after the shift into the complex plane, we will get the same solution to this integral as long as the boundary conditions remain the same.

The coordinates

Coordinates of space (x) and time (t) can be translated into coordinates of the boost parameter (η) and rho (ρ) by the following equations:

$$x = \rho \cosh(\eta)$$

$$t = \rho \sinh(\eta)$$

To shift the graph slightly into the *i* dimension, add *i*ε to ρ, where ε represents a very small number.

$$\rho \rightarrow \rho + i\varepsilon$$

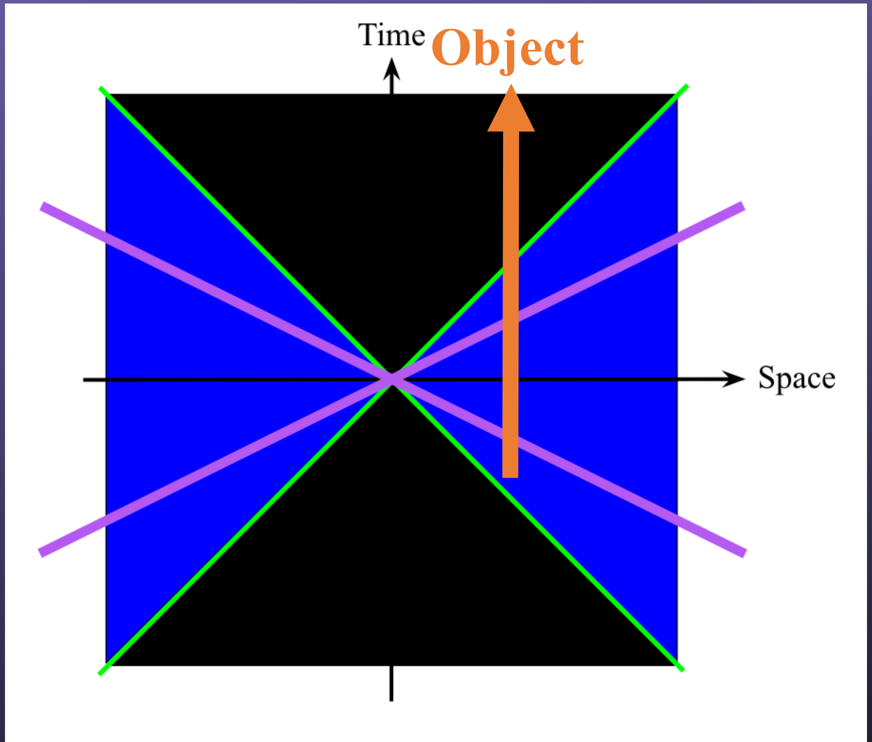
To maintain proper boundary conditions, make ε dependent on ρ such that

$$\lim_{\rho \to \infty} \varepsilon = 0$$

Results of the Shift (Part 1)

In real space (before the shift):

The worldline of an object in freefall will leave the right quadrant and enter the top quadrant after a finite amount of proper time.



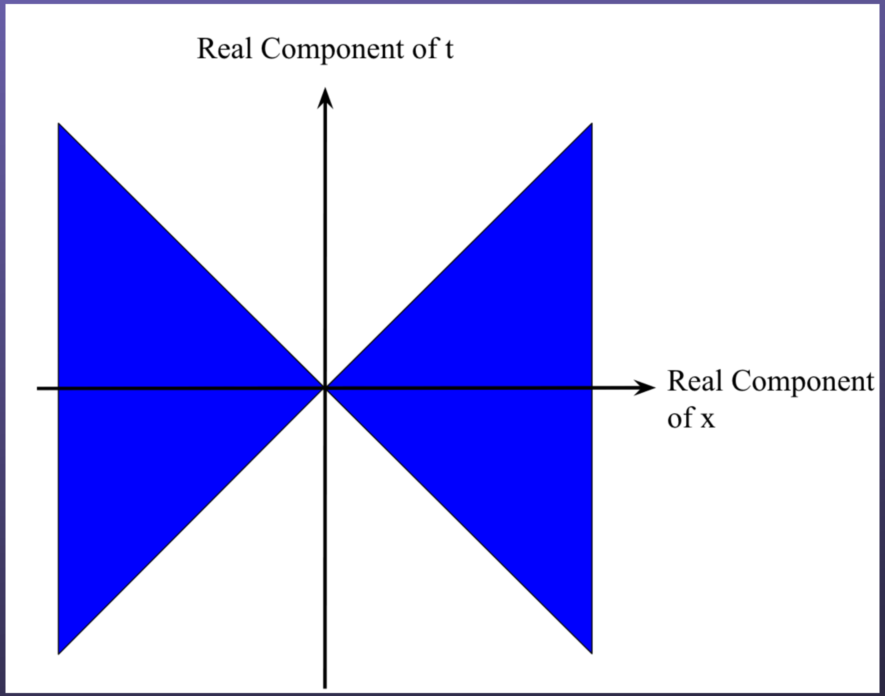
Results of the Shift (Part 2)

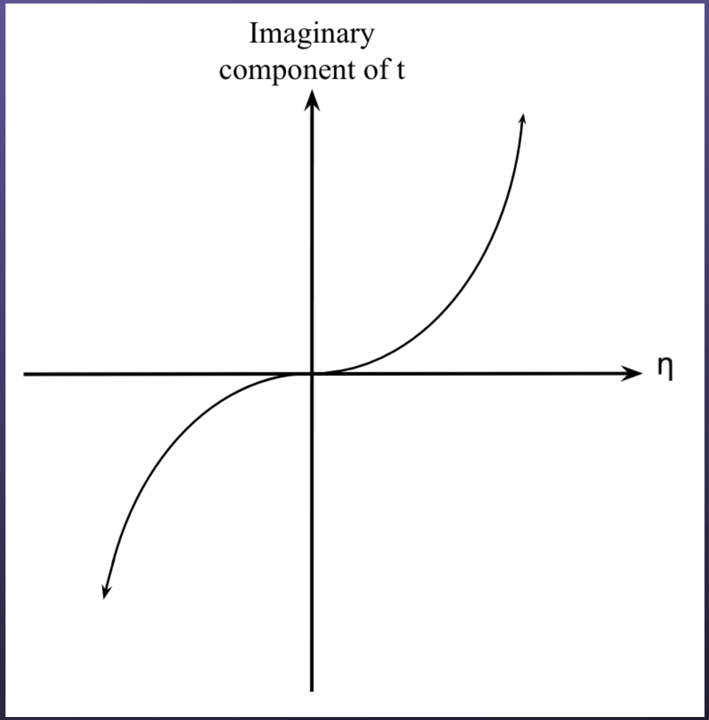
<u>In complex space (after the shift):</u>

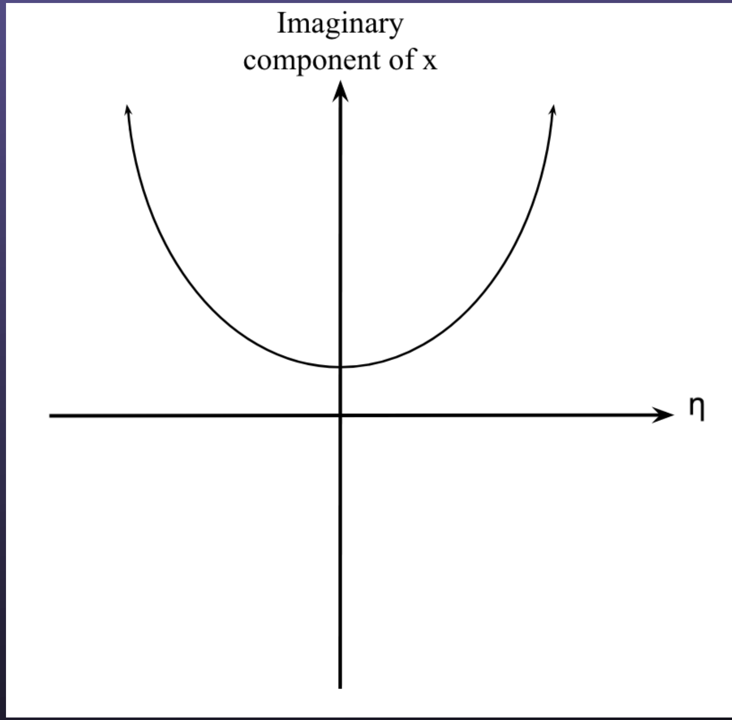
The worldline of this same object will never leave the right region.

This is because the range of the imaginary part of t becomes [-∞, ∞] in the right region.

Additionally, the range of the imaginary part of x becomes [ε, ∞] in the right region.



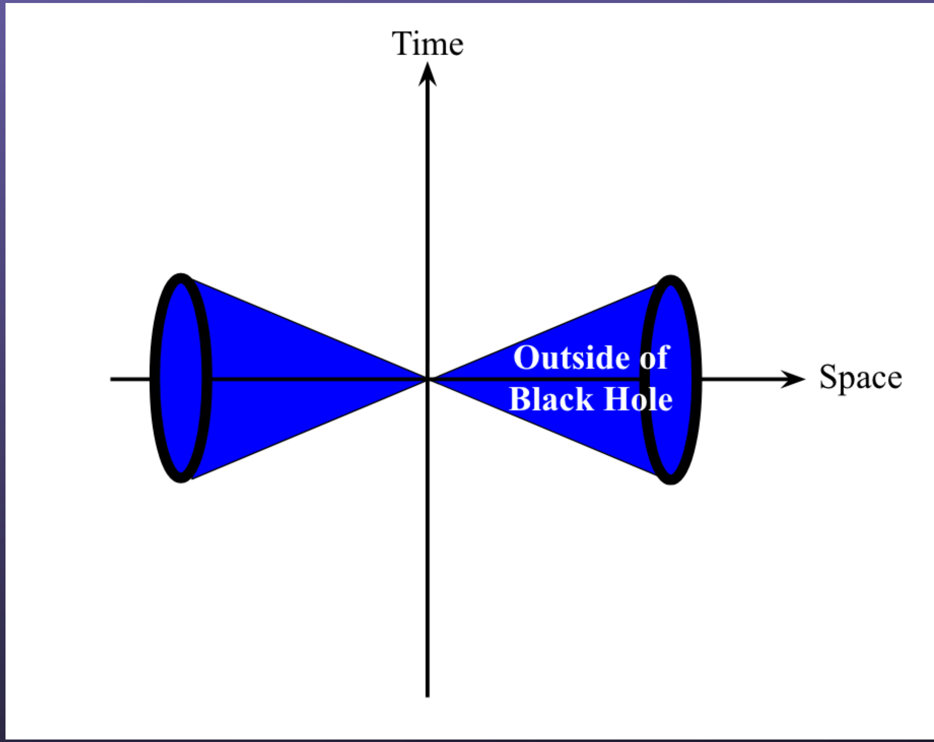




[ρ is fixed at *i*ε in the above two graphs]

Results of the Shift (Part 3)

Therefore, we can conclude that the double cone spacetime gives a complete picture of what happens to a traveling object without any modifications (such as the addition of the top and bottom regions).



What does this achieve?

This gives us a better understanding and confirmation of completeness of the double cone spacetime created by Douglass Stanford and Steve Shenker.

This double cone spacetime is an essential part of a calculation which will determine the quantum properties of black holes and ultimately help show how quantum mechanics and general relativity fit together.

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