Uncovering the Secrets of Gravity: Towards an Interpretation of the RT Formula without Holography

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Abstract

The AdS/CFT correspondence is one of the few tools that exists to analyze nonperturbative effects in quantum gravity. Specifically, the duality allows for the computation of the entropy of a subregion of the CFT through a relation to an extremal surface in the bulk. However, whether it can be interpreted as an entropy for the quantum gravity theory is unknown. Recent work has shown in the case of two-boundary states constructed from the gravitational path integral, a von Neumann algebra can be defined and a notion of entropy can be computed using said mathematical structure. We generalize this to the one boundary case, which comes with the added difficulty that the resulting von Neumann algebras are type III and thus have no trace operation. To circumvent this issue we propose a regulated form of entanglement entropy and connect this to a replica trick to circumvent the type III nature of the von Neumann algebra.

1 Introduction

Finding a quantum theory of gravity remains one of the great unsolved problems in physics. One of the major developments in the field was the discovery that certain quantum gravity theories are dual to a class of quantum field theories known as conformal field theories (CFT). This duality exists such that certain observables in the quantum gravity theory are related to other observables in the CFT. One important aspect of the so-called AdS/CFT correspondence is that the CFT lives on the boundary of the dual quantum gravity theory, providing some geometric intuition to the relation.

One of the outstanding issues of quantum gravity is a lack of understanding surrounding gravitational entropy. The AdS/CFT correspondence can perhaps help in this regard due to the relation between the entropy of a subregion of the boundary CFT and the area of an extremal surface within the bulk [1]. The formula that relates these is known as the Ryu-Takayanagi (RT) formula, which was later generalized to the quantum extremal surface (QES) formula [2].

Since we care about the entropy of the quantum gravity theory, one may wonder what the bulk interpretation of the entropy computed by the RT formula may be. In a general quantum gravity theory, this is not currently well understood. The QES formula can be derived using a gravitational path integral [3], which implies this is something worth studying, but the specifics are unknown.

One of the difficulties in understanding gravitational entropy is that the microstates are a mystery, which makes calculating this observable difficult. However, looking at the von Neumann algebra of a system and calculating its entropy is an alternative approach to constructing a trace operator and density matrix to find the von Neumann entropy of a system. Therefore, von Neumann algebras are a tool which can be used as a method of calculating the entropy of a subregion of the bulk theory, which then can be compared to the RT formula. After a brief review of path integrals in section two, the construction of the von Neumann algebra of a subregion of the bulk is tackled in section three. Then, in section four, entropy considerations are addressed, and finally, section five contains a discussion of results.

2 Gravitational Path Integrals

2.1 Path Integrals in Quantum Mechanics

It is known that to compute the probability amplitude between two given states in single particle quantum mechanics, the following equation holds [4]:

$$\langle x_i, t_i | x_f, t_f \rangle = \int_{x(t_i)=x_i, x(t_f)=x_f} \mathcal{D}x e^{i \int_{t_i}^{t_f} dt\, L(x,\dot{x})} \tag{1}$$

Where the right hand side is what is known as a path integral, a sum over the set, $\mathcal{D}x$, of all possible trajectories a classical particle could take with fixed boundary conditions, where each is weighted by how close it is to the path which minimizes the classical action. For more details (and on what follows in this subsection), see chapter 2 of [4]. However, probability amplitudes are not the only thing path integrals can be used for: we can also compute partition functions. To see this, is is useful to rewrite the probability amplitude in (1) using the time evolution operator:

$$\langle x_i, t_i | x_f, t_f \rangle = \langle x_i, t_i | e^{-iH\Delta t} | x_f, t_i \rangle \tag{2}$$

With this in mind, we can work backwards from the definition of the partition function in thermodynamics and make a connection to (2):

$$\begin{aligned} Z(\beta) = \mathrm{Tr}(e^{-\beta H}) &= \sum_k \langle k | e^{-\beta H} | k \rangle \\ &= \int dx_f \, \langle x_f, t_i | e^{-\beta H} | x_f, t_i \rangle \end{aligned} \tag{3}$$

Notice how similar the bra-ket in the right hand side of the final equality is to the right hand side of (2), but with $\beta = i\Delta t$ and our initial and final states existing at the same spatial point. In other words, the partition function is a sum over all possible trajectories satisfying periodic boundary conditions at every possible start/end location, where the time coordinate is multiplied by a factor of i. We are now ready to write the partition function in quantum mechanics as a path integral:

$$Z(\beta) = \int_{PBC} \mathcal{D}x e^{\int_0^\beta d\tau\, L(x,\dot{x})} \quad (4)$$

While it may seem strange at first that path integrals can be used to calculate two different physical quantities, it is important to keep in mind the geometric interpretations of the trajectories in both cases. When calculating transition amplitudes, the path integral sums over trajectories that look like curves with unique start and end points. In the case of the partition function, the start and end points are the same, which one can think of as gluing the start and end points of the path integral for the transition amplitude. To sum over all periodic boundary conditions, we must then sum over all possible places where the two ends of a trajectory could be sewed together. Therefore, the difference between calculating transition amplitudes and partition functions comes down to fixing certain boundary conditions and specifying which are summed over.

Lastly, states can be prepared by the path integral by only specifying the initial boundary condition, equivalent to summing over all states that start at a fixed point and end at any other point. This is given by the equation:

$$|\psi\rangle = \int_{x(t_i)=x_i} \mathcal{D}x e^{\int_0^\beta d\tau\, L(x,\dot{x})} \quad (5)$$

2.2 Path Integrals in Gravity

2.2.1 The Intuitive Approach

With the prior section in mind, the first perspective on the gravitational path integral will look something like an adaption of (4) to the gravitational case. There are a number of technical difficulties associated with this [5], which we will for the most part avoid by adopting a new perspective of the path integral in the next section. There are a few components of the path integral we will modify for quantum gravity, which will be obvious by its change in appearance:

$$\zeta(M) = \int \mathcal{D}\phi\, e^{-S[\phi]} \quad (6)$$

In the previous section, trajectories are paths between spatial states. In gravity, states are given by spacetime geometries, and a trajectory is a transition from one spacetime configuration to another. Formally, this can be thought of as a change from one induced boundary metric to another. The reason the boundary metric specifies our configuration is related to the action in (6). S is the Einstein-Hilbert action with the Gibbons-Hawking-York boundary term. It turns out that gravitational systems with a boundary can be fully described by boundary quantities, which is very convenient for the path integral.

2.2.2 The Axiomatic Approach

The reason for the change in notation in (6) on the left hand side is because, for the purposes of this project, the gravitational path integral takes on a more abstract interpretation. In the previous section, the partition function was a function of β, meaning that for every possible choice of this value, the path integral gives a number. In the gravitational case, due to the geometric interpretation discussed above, the path integral is now a function of source manifolds (boundary manifolds with boundary conditions for a field ϕ). So the path integral can be represented more formally as a function:

$$\zeta : X^d \to \mathbb{C} \quad (7)$$

Where X^d is the set of source manifolds. Now we will give (7) a set of axioms that we assume hold for a UV-complete gravitational path integral. For the rest of the paper, this and this alone will be the definition of the path integral we consider. While it might seem strange to throw out all the physical intuition of the previous sections for this abstract definition, remember that this intuition is laden with technical issues. For a longer discussion on where these axioms come from, see [6].

Axiom 1 (Finiteness) *For any smooth manifold M with smooth source fields in X^d, $\zeta(M) < \infty$*

Axiom 2 (Reality) *If $M \in X^d$, then $M^* \in X^d$. Furthermore, $[\zeta(M)]^* = \zeta(M^*)$*

Axiom 3 (Reflection Positivity) *Let $n \in \mathbb{Z}^+$, $\gamma_K \in \mathbb{C}$. If $M = \sum_{I,J=1}^{n} \gamma_I^* \gamma_J M_{I,J}$, where $M_{I,J}$ denotes a partition of M into two manifolds-with-boundary N_I^*, N_J [6], $\zeta(M)$ is a non-negative real number.*

Axiom 4 (Continuity) *Let $M_\epsilon \in X^d$ have a region diffeomorphic to an orthogonal cylinder source manifold C_ϵ [6]. $\zeta(M_\epsilon)$ must be continuous in ϵ.*

Axiom 5 (Factorization) *Let $M_1, M_2 \in X^d$. Then $\zeta(M_1 \sqcup M_2) = \zeta(M_1)\zeta(M_2)$*

Axioms 1-3 were discussed in [7] as necessary even for a gravitational path integral in the form of (6). The utility of axiom 3 will become apparent when constructing sectors of the quantum gravity Hilbert space in the next section. It seems reasonable to expect the path integral to be continuous under slight deformations of boundary conditions, hence axiom 4. Lastly, axiom 5 is physically reasonable and is compatible with the idea of α sectors [8].

2.3 Construction of Quantum Gravity Hilbert Space

This section closely follows the analogous construction in [6]. Using (7) (along with axioms 1-5) we can prepare states using ideas similar to what was described above for the quantum mechanical case. This is known as cutting open the path integral [8]. Intuitively, this can be thought of as cutting an $M \in X^d$ into two pieces, N_1 and N_2 (the states), such that their inner product is given by $\zeta(M)$. There turns out to be a few technical issues associated with constructing a Hilbert space in this manner, which the remainder of this section is dedicated to discussing.

In general, if open sets of N_1 and N_2 contain symmetries, there are multiple ways to glue them back together into a manifold that may be different than M. Thus, it is useful to think of N_1 and N_2 with boundaries $\partial N_1 = \partial N_2$ having the additional structure of a labelling of points on the boundary. Then, the inner product on the Hilbert space glues the two manifolds-with-boundary according to this labelling, eliminating the possibility of multiple gluings for the same manifolds-with-boundary.

But there is another problem: there is no guarantee through the gluing that the boundary fields will be continuous. To circumvent this issue we construct a sector of the quantum gravity Hilbert space where manifolds-with-boundary have the additional requirement of having two boundary dimensions. We define $H_{\partial N}$ as the sector of the quantum gravity pre-Hilbert space with states from the space of 2 boundary dimension manifolds $Y^d_{\partial N}$ and an inner product defined by:

$$\langle N_1|N_2\rangle := \zeta(M_{N_1^* N_2}) \tag{8}$$

Where $M_{N_1^* N_2}$ denotes the manifold M glued together by the two manifolds-with-boundary N_1^* and N_2. After completing the quotient of the pre-Hilbert space with respect to the space of null vectors $\mathcal{N}_{\partial N}$, we arrive at $\mathcal{H}_{\partial N}$, the sector of the quantum gravity Hilbert space associated with manifolds-with-boundary with two boundary dimensions.

3 Construction of von Neumann Algebra

Before constructing an operator algebra which acts on $\mathcal{H}_{\partial N}$, we introduce the concept of a von Neumann algebra and introduce some facts useful for what will follow.

3.1 Basic facts related to von Neumann Algebras

The first ingredient in a von Neumann Algebra is an algebra of bounded operators, so to begin, we explore this concept.

Definition 1 *A bounded operator acting on a Hilbert space $\mathcal{H}$ is a map*

$$\mathcal{O} : \mathcal{H} \to \mathcal{H} \tag{9}$$

Such that:

$$\|\mathcal{O}\,|\psi\rangle\,\| \leq C\|\,|\psi\rangle\,\| \tag{10}$$

where $|\psi\rangle \in \mathcal{H}$ and C is a scalar.

Bounded operators are important because the algebra they form represents the observables of a quantum field theory. So the next step is to define this algebra:

Definition 2 *An algebra of bounded operators $\mathcal{A}$ is a topological vector space of operators which are continuous, linear, and bounded over a field K equipped with an operation given by:*

$$\mathcal{O}_1 \circ \mathcal{O}_2\,|\psi\rangle = \mathcal{O}_1(\mathcal{O}_2(|\psi\rangle)) \tag{11}$$

Where $\mathcal{O}_1, \mathcal{O}_2 \in \mathcal{A}$ and $|\psi\rangle \in \mathcal{H}$.

Self-adjoint operators play a key role in the theory of quantum mechanics, and thus next we define an operation which generalizes this notion, known as a *-operation

Definition 3 *A *-operation over an algebra of operators $\mathcal{A}$ is a map $^* : \mathcal{A} \to \mathcal{A}$ such that:*

$$(\mathcal{O}_1 + \mathcal{O}_2)^* = \mathcal{O}_1^* + \mathcal{O}_2^* \tag{12}$$

$$(\mathcal{O}_1\mathcal{O}_2)^* = \mathcal{O}_2^*\mathcal{O}_1^* \tag{13}$$

$$\mathbb{1}^* = \mathbb{1} \tag{14}$$

$$(\mathcal{O}^*)^* = \mathcal{O} \tag{15}$$

Where $\mathcal{O}_1, \mathcal{O}_2 \in \mathcal{A}$ and $\mathbb{1}$ is the identity operator

Lastly, the *-algebra defined above (the algebra of operators with the *-operation) needs to be closed under the weak operator topology, which defines a notion of taking limits.

Definition 4 *A *-algebra $\mathcal{A}$ is closed under the weak operator topology if a sequence of operators $\mathcal{O}_1, \mathcal{O}_2, ..., \mathcal{O}_n$ contained in $\mathcal{A}$ converges to an operator $\mathcal{O} \in \mathcal{A}$. Specifically, the matrix elements associated with the operator sequence converge: $\lim_{n\to\infty} \langle\psi|\mathcal{O}_n|\phi\rangle = \langle\psi|\mathcal{O}|\phi\rangle$ If this condition holds for a *-algebra, it is a von Neumann algebra.*

There are three types of von Neumann Algebra factors (algebras with a trivial center), and any von Neumann Algebra can be written as a direct sum of these factors. The definitions of the types of von Neumann algebras are related to projections operators $P \in \mathcal{A}$. For our purposes we will not explicitly give these definitions and instead discuss what the trace operation looks like on the different types of von Neumann algebras. For a more rigorous treatment, see [9]. A trace is a linear map satisfying $\mathrm{Tr}(U^\dagger \mathcal{O} U) = \mathrm{Tr}(\mathcal{O})$ for $\mathcal{O}$ in $\mathcal{A}$ and U is a unitary operator in $\mathcal{A}$. Additionally, there are three other properties that a trace operation may or may not have which will help us distinguish between the cases:

1. Faithfulness: $\mathrm{Tr}(\mathcal{O}) = 0$ if and only if $\mathcal{O} = 0$
2. Semifiniteness: For all positive operators $\mathcal{O}_p$ there exists another (nonzero) positive operator $\mathcal{O}'_p$ such that $\mathcal{O}_p - \mathcal{O}'_p > 0$ and $\mathrm{Tr}(\mathcal{O}'_p) < \infty$
3. Normality: Let $\{\rho_\alpha\}$ be a bounded sequence of positive operators. Then $\rho = \sup_\alpha \rho_\alpha$ implies $\mathrm{Tr}(\rho) = \sup_\alpha \mathrm{Tr}(\rho_\alpha)$

Every trace discussed below is assumed to be faithful. Type I algebras have a normal semifinite trace that, after normalization, is greater than or equal to 1 for nonzero projections. Type II algebras have a normal semifinite trace where $\mathrm{Tr}(P) \in [0, 1]$ or $[0, \infty]$. Type III algebras have no normal semifinite trace (meaning $\mathrm{Tr}(P) = 0$ or ∞).

3.2 The von Neumann Algebra of One-Boundary States

3.2.1 Defining the Surface Algebra

In [6], the algebra constructed only acts on the sector of $\mathcal{H}_{\partial N}$ where the states (manifolds-with-boundary from a sliced path integral) consisted of two identical and disjoint pieces. We instead consider the sector of the Hilbert space $\mathcal{H}_1 = \mathcal{H}_{L\cup R}$, the Hilbert space of states with only one boundary. We need to divide the boundary into two pieces, analogous to having two disjoint boundaries.

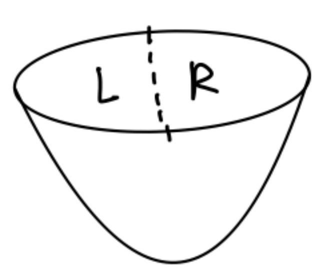

Figure 1: A visual representation of a one boundary state, with a left and right region mimicking the two boundary case.

Just as in the original calculation, an operator can be associated with each $N \in Y^d_{L\cup R}$, where $Y^d_{L\cup R}$ is the space of one-boundary source manifolds divided into a left and right side, as seen in figure 1. Multiplication on the left surface algebra A_L can be defined as:

$$a \cdot_L b \quad a, b \in Y^d_{L\cup R} \tag{16}$$

Where this operation glues the left boundary of b to the right boundary of a. The right surface algebra can be defined similarly, gluing the left boundary of b to the right boundary of a. Since we have already specialized to work in two boundary dimensions, we avoid issues of unmatched extrinsic curvature and specialize to the case where the left and right boundaries are identical, denoted $Y^d_{B\cup B}$. It would be interesting to try and recover the same result while relaxing these two assumptions, but in higher dimensions it seems difficult to have a well-defined gluing operation. To summarize, the left surface algebra A_L is a pair $(Y^d_{B\cup B}, \cdot_L)$ as defined above.

3.2.2 Representation of Surface Algebras on $\mathcal{H}_{L\cup R}$

Having constructed a surface algebra, we can now define a representation that is the final step before getting the von Neumann algebra. To begin, consider the action $\hat{a}_L$ of an operator acting on the pre-Hilbert space $H_{B\cup B}$:

$$\hat{a}_L = |a \cdot_L b\rangle \ \forall |b\rangle \in H_{B\cup B} \tag{17}$$

where $a \in Y^d_{B\cup B}$. Hence, $\hat{a}_L$ acts on $|b\rangle$ by gluing the two surfaces together just as discussed in the previous section. We defer to [6] for the proof that $\hat{a}_L$ is well defined on the pre-Hilbert space. Since the one boundary case lacks a trace inequality, we assume that $\hat{a}_L$ is well defined after taking the quotient by the nullspace and is properly extended on the full Hilbert space $\mathcal{H}_{B\cup B}$. This is well motivated by the fact that in many systems there is only a single element of the nullspace $\mathcal{N}_L$. After this, we are now left with a representation $\hat{A}_L$ acting on $\mathcal{H}_{B\cup B}$. Similarly, starting with $\hat{a}_R$, we can define the right representation $\hat{A}_R$ using the multiplication $\cdot_R$.

3.2.3 Calculating the von Neumann Algebra

To define the left and right von Neumann algebras of one boundary operators, simply take the quotient of the representation constructed above, netting us $\hat{A}_L/\mathcal{N}_L$, and then take its closure in the weak operator topology. Since we assumed $\hat{a}_L$ is well defined on the quotient space, the resulting algebra $\mathcal{A}_L$ (and $\mathcal{A}_R$) is what we would like to prove is a von Neumann algebra. The largest obstacle in this is proving that $\mathcal{A}_L$ only contains bounded operators. There is no easy way to do this without a sensible trace operation, and since we don't expect to be able to define one on $\mathcal{A}_L$, this makes the task quite difficult. So, instead, we will further assume that $\mathcal{A}_L$ is an algebra of bounded operators. It is possible to define an involution corresponding to the adjoint of an operator on the surface algebra (as done in [6]), which can be naturally extended to $\mathcal{A}_L$. Since we have already taken the closure in the weak operator topology, this shows that (with proper assumptions) $\mathcal{A}_L$ is a von Neumann algebra. Due to our construction matching that of a von Neumann algebra of a subregion of a quantum field theory, we expect this algebra to be type III, with no semifinite and normal trace operation existing on our space.

4 Entropy

Computing the entanglement entropy of operators will be difficult due to the type-III nature of $\mathcal{A}_L$ and $\mathcal{A}_R$: without a sensible trace operation finding a notion of entropy seems impossible. However, it turns out our heavy reliance on algebraic quantum field theory will allow us to use the split property [10] to define a regulator which recovers a sensible notion of entropy.

4.1 The Split Property

We begin with a quick review of the split property. In AQFT, the independence of regions of spacetime goes beyond the Einstein causality condition:

$$[\mathcal{A}_2, \mathcal{A}_3] = 0 \tag{18}$$

Where $\mathcal{A}_2$, $\mathcal{A}_3$ are von Neumann algebras of observables for regions of spacetime 2 and 3, given below:

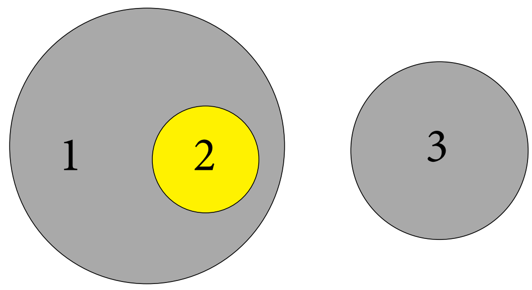

Figure 2: An example of the split property in quantum field theory, where the independence of the two regions 2 and 3 can be demonstrated with a collared region 1.

Instead, assuming there exists a collar region (labelled 1 above) surrounding region 2, one can find a type I factor that splits the algebra $\mathcal{A}_2$ of the algebra of the larger region $\mathcal{A}_1$:

$$\mathcal{A}_2 \subset \mathcal{N} \subset \mathcal{A}_1 \tag{19}$$

Where $\mathcal{N}$ is the type-I factor. With this setup it is possible to find a map which implements the isomorphism

$$\mathcal{A}_1 \vee \mathcal{A}_3 \cong \mathcal{A}_1 \otimes \mathcal{A}_3 \tag{20}$$

Which establishes the independence of the two regions. For more details, see [10].

4.2 Utility of the Split Property in Quantum Gravity

As a qualification for causality, there are good reasons to expect that the split property requires some modification in the gravitational setting. But this is not our use: instead, we introduce a regulator to setup a system on a one-boundary state where the split property applies. Then, using the existence of a type I factor, an algebra with a trace, we can calculate an entropy-like quantity according to section 7 of [11]. Suppressing bulk dimensions, the visual of what this looks like is as follows:

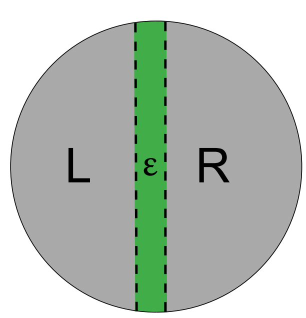

Figure 3: A proposed regulator for one boundary states, where the green region contains parts of both the left and right regions.

Where, without loss of generality, we take the left region to be the one "collared" by the finite gap. Clearly, as $\epsilon \to 0$, we get our initial state back. It is immediately clear that this also solves the biggest issue with the original setup: the shared middle boundary between the two regions. From here on out, we refer to the construction of section 7 of [11], offering some speculative ideas as to how this entropy can actually be computed. This puts our knowledge of entropy in this setting on foot with quantum field theory, which while incomplete, is still progress.

It turns out that the type-I factor $\mathcal{N}$ is not unique, but there is a cannonical choice related to each state ψ, $\mathcal{N}_\psi$. One can then define a quantity known as the splitting entropy on this type I factor using the normal von Neumann entropy formula:

$$S_R = \text{Tr}(\rho_\psi log \rho_\psi) \tag{21}$$

Where ρ_ψ is the density matrix of a state derived from the canonical choice of N_ψ. The next step in this conjecture is that this quantity is related to the regulated entanglement entropy of the state:

$$S_{EE}^{(\epsilon)} = \frac{1}{2} S_R = \frac{1}{2} S_{VN}(\mathcal{N}_\psi) \tag{22}$$

Finally, this can then all be related to the replica trick described in section 4 of [11], which links us back to our original story of path integrals.

5 Discussion

After a short review of path integrals, the space of states of one boundary (in two boundary dimensions) was constructed. This is a previously unexplored sector of the quantum gravity Hilbert space. Next, after dividing the boundary into two pieces, a von Neumann algebra was constructed in two boundary dimensions. Issues with matching extrinsic curvature seem like they could lead to problems in higher dimensions, where new path integral axioms may need to be introduced in order to fix this problem. Additionally, two more assumptions were made in defining the von Neumann algebra in comparison to [6]: boundedness of operators and, without loss of generality, the extension of the left surface algebra representation $\hat{a}_L$ after taking a quotient by null states. Finally, after introducing the split property, we discuss a conjectured regulator for entanglement entropy that could help with the type-III nature of the algebra associated with one-boundary states.

Despite being a conjecture, this puts our understand of gravitational entropy (in this sector of the Hilbert space) on par with that of quantum field theory. This represents an advance in understanding, but there is still much work to be done. Proving boundedness of operators would be useful and seems possible, but more interesting would be to prove the conjecture discussed in the final section of the paper relating replica tricks, entropy, and the split property. This would advance knowledge of both QFT and quantum gravity and introduce in a more rigorous way a new class of replica tricks that could be studied in quantum gravity (although any replica trick is far from rigourous for now).

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