Investigating the RK Point of a Hamiltonian with Fractal Symmetry

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UC SANTA BARBARA PHYSICS

NSF

Condensed Matter Theory

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

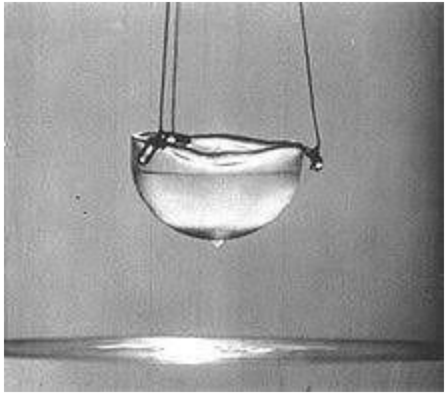
Study of many body quantum systems, quantum matter

Motivation: Translates to descriptions of macroscopic objects of interest for modern technology

→ Superfluids

→ Superconductors

How can we analyze these systems?



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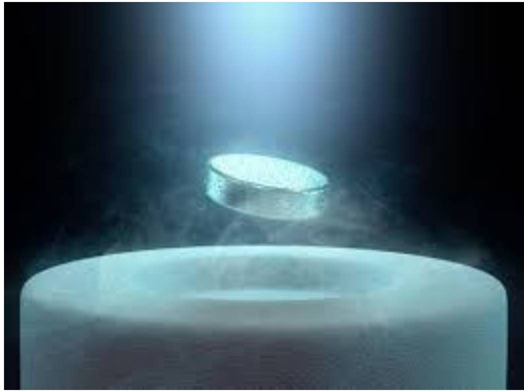

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Analyzing Large Quantum Systems

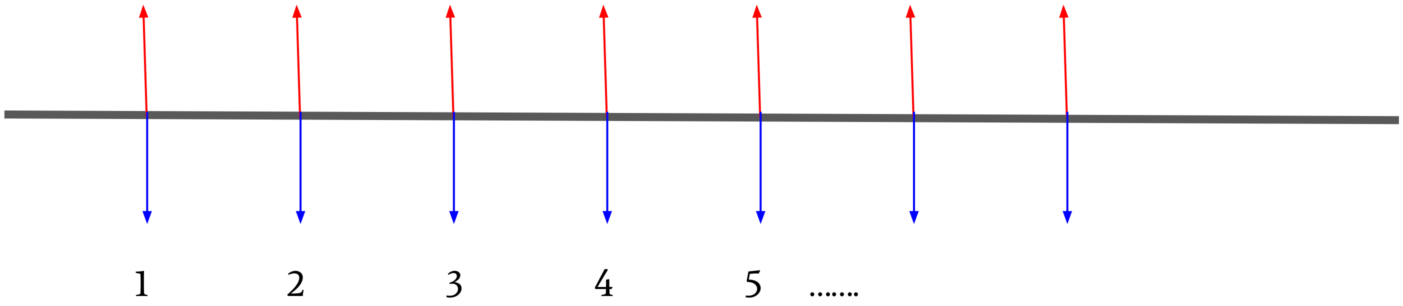
Energetics of Quantum System Given by a Hamiltonian

Correlated States

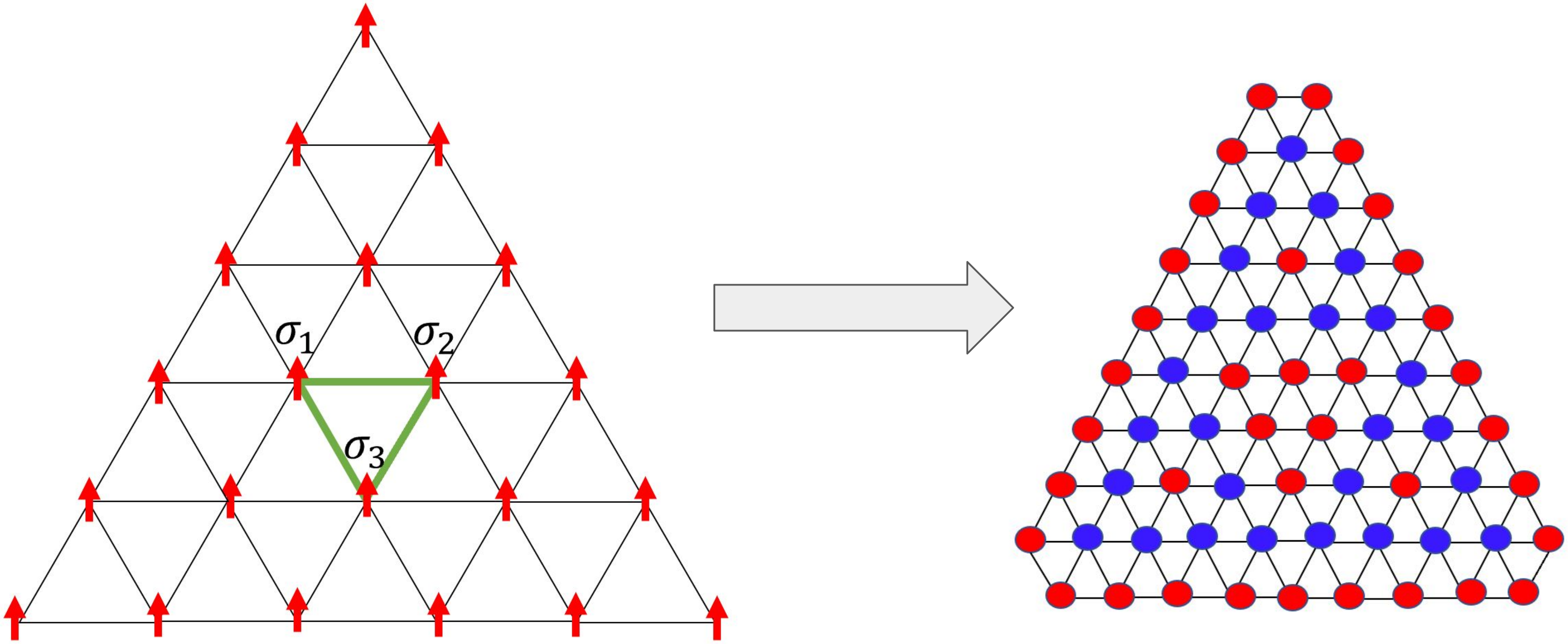
Symmetries → Unitary Operators!

$$[\mathcal{U}, H] = 0$$

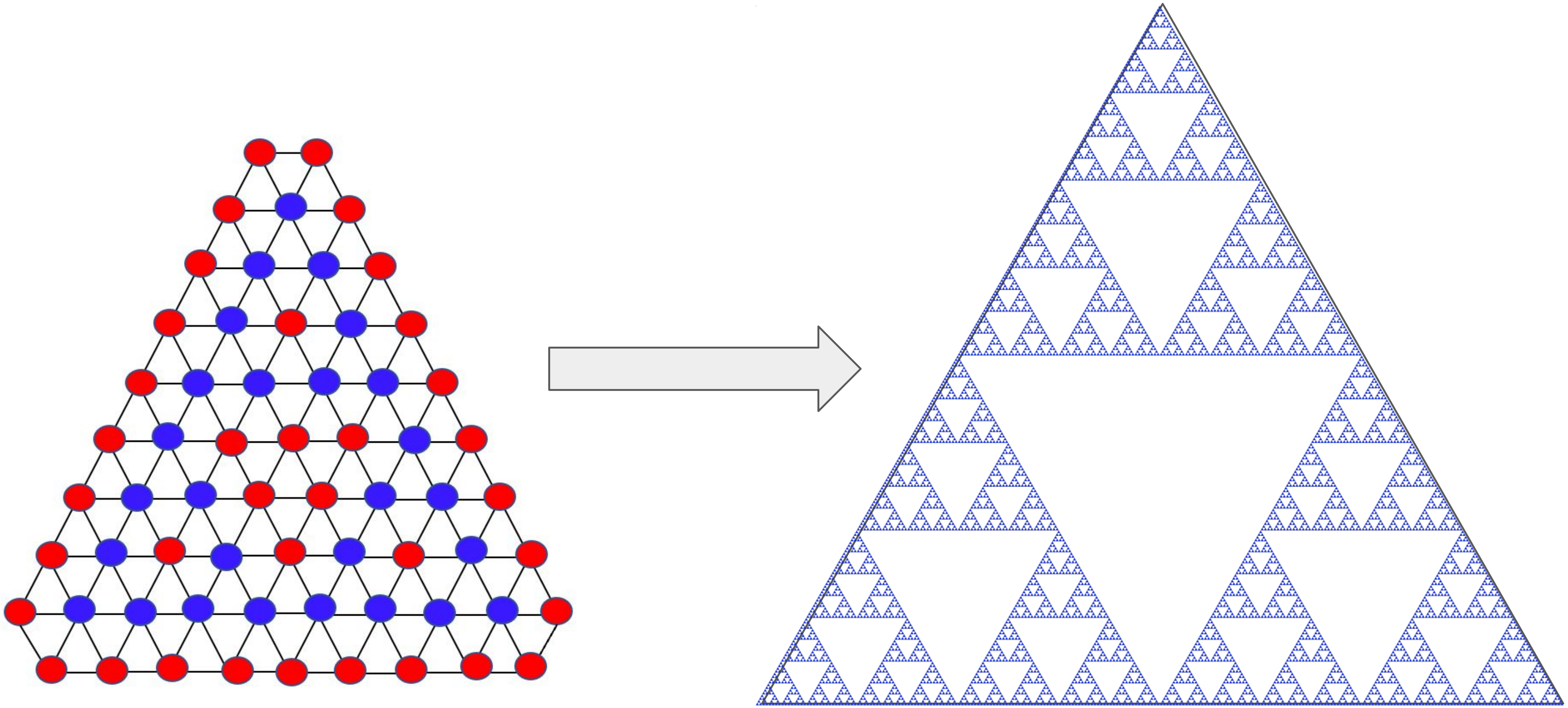
Spontaneous Symmetry Breaking



Fractal Symmetries



Fractal Symmetries



https://upload.wikimedia.org/wikipedia/commons/thumb/4/45/Sierpinski_triangle.svg/1200px-Sierpinski_triangle.svg.png

The Hamiltonian

$$H = t \sum_{\nabla} \left[|\phi_{\nabla}^{\uparrow}\rangle - |\phi_{\nabla}^{\downarrow}\rangle \right] \left[\langle\phi_{\nabla}^{\uparrow}| - \langle\phi_{\nabla}^{\downarrow}| \right] + h \sum_{\nabla} \left[|\phi_{\nabla}^{\uparrow}\rangle\langle\phi_{\nabla}^{\uparrow}| + |\phi_{\nabla}^{\downarrow}\rangle\langle\phi_{\nabla}^{\downarrow}| \right]$$

The Hamiltonian

$$H = \boxed{t} \sum_{\nabla} \boxed{\left[|\phi^{\uparrow}_{\nabla}\rangle - |\phi^{\downarrow}_{\nabla}\rangle \right] \left[\langle\phi^{\uparrow}_{\nabla}| - \langle\phi^{\downarrow}_{\nabla}| \right]} + \boxed{h} \sum_{\nabla} \boxed{\left[|\phi^{\uparrow}_{\nabla}\rangle\langle\phi^{\uparrow}_{\nabla}| + |\phi^{\downarrow}_{\nabla}\rangle\langle\phi^{\downarrow}_{\nabla}| \right]}$$

Constants

Projection Operators
(All Up and All Down Triangles)

Varying the constants allows for
different symmetries

The Hamiltonian

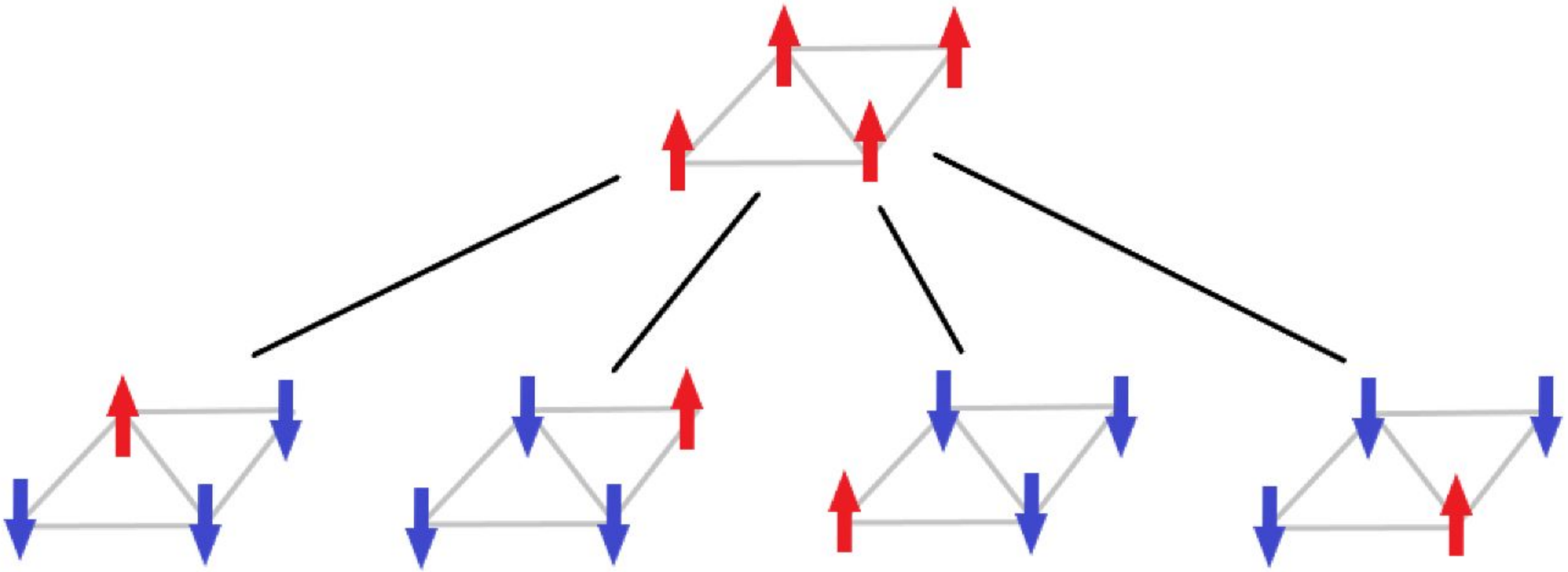
$$H = t\sum_{\nabla}\left[|\phi^{\uparrow}_{\nabla}\rangle - |\phi^{\downarrow}_{\nabla}\rangle\right]\left[\langle\phi^{\uparrow}_{\nabla}| - \langle\phi^{\downarrow}_{\nabla}|\right] + h\sum_{\nabla}\left[|\phi^{\uparrow}_{\nabla}\rangle\langle\phi^{\uparrow}_{\nabla}| + |\phi^{\downarrow}_{\nabla}\rangle\langle\phi^{\downarrow}_{\nabla}|\right]$$

RK Point: h = 0

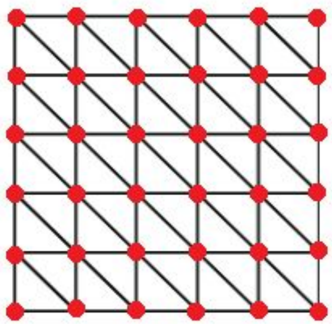
RK Points are a type of critical point in condensed matter systems

Allows for a special type of ground state:
Superposition of states reachable by “local flips”

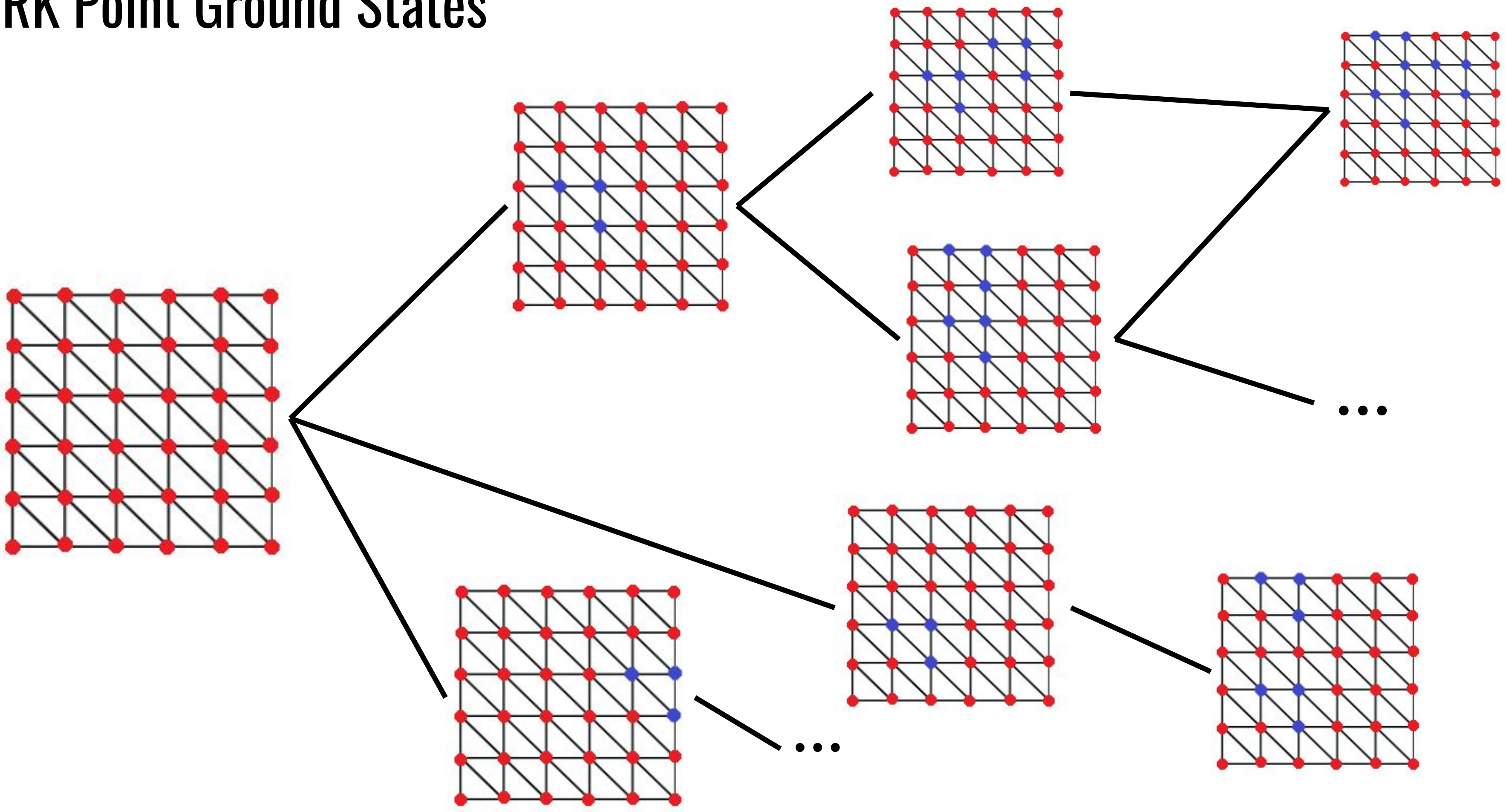
RK Point Ground States



RK Point Ground States



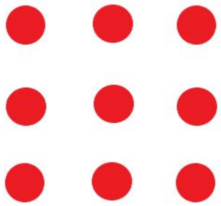
RK Point Ground States



Eigenstates of the Hamiltonian

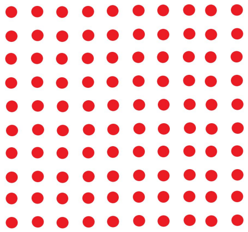
"Hilbert space is a big place"
-Carlton Caves

3 x 3 Lattice → 512 Possible States



Can Use Exact Diagonalization
Use Sparse Matrix Techniques

10 x 10 Lattice → ~10^{30} Possible States

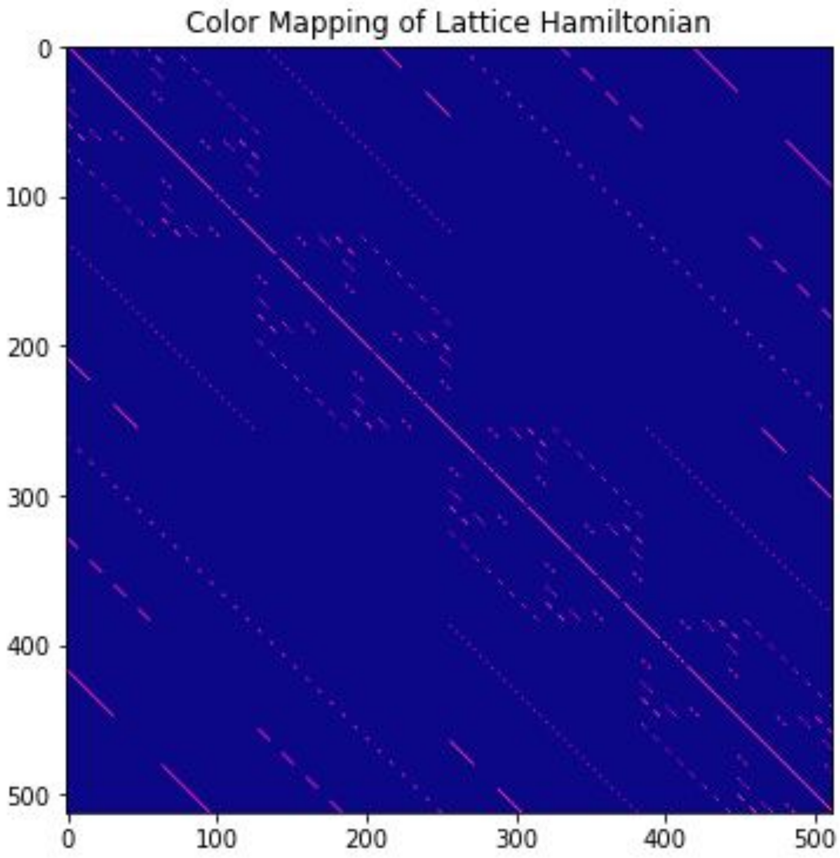


Impossible to Calculate Exactly
Use Monte Carlo Simulation

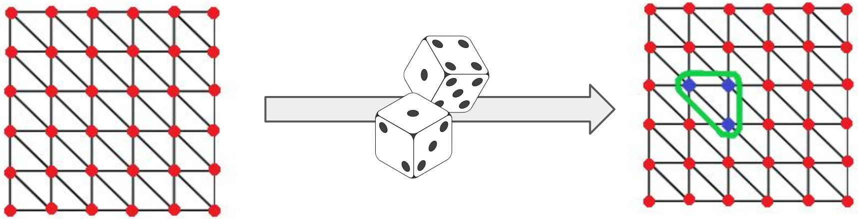
Exact Diagonalization and Sparse Matrices

Blue: Zero Matrix Elements

Pink: Non-Zero Matrix Elements



Monte Carlo Method



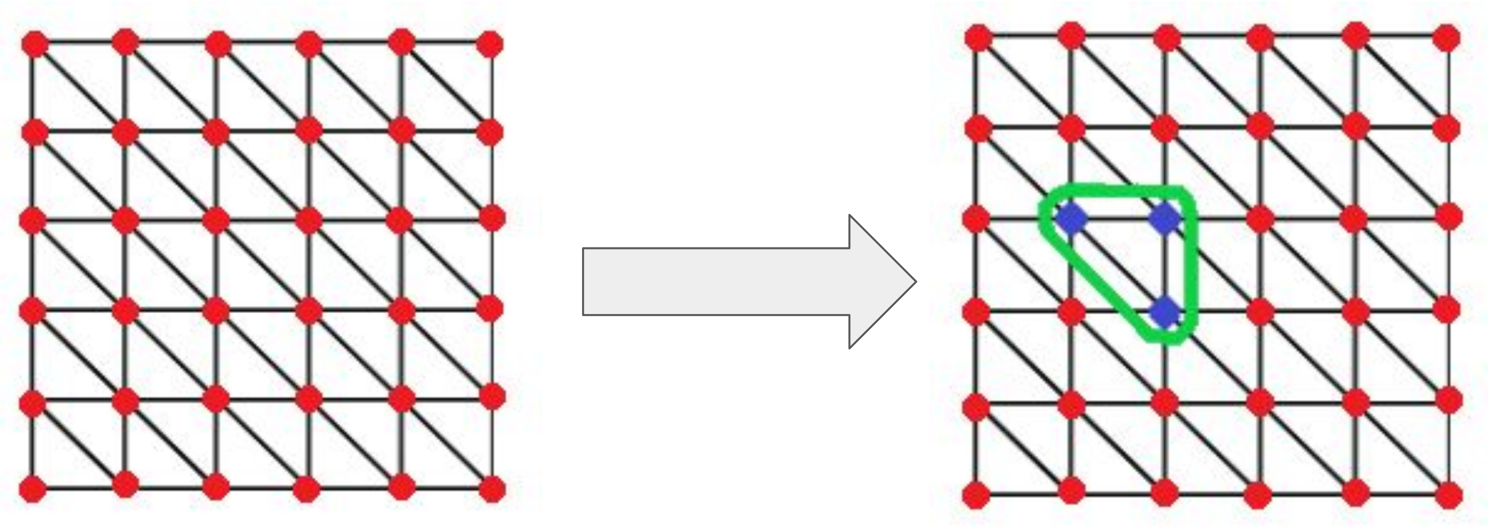
-Uses a random process to flip through states and measure correlations

-Random sampling to obtain information about the ground states

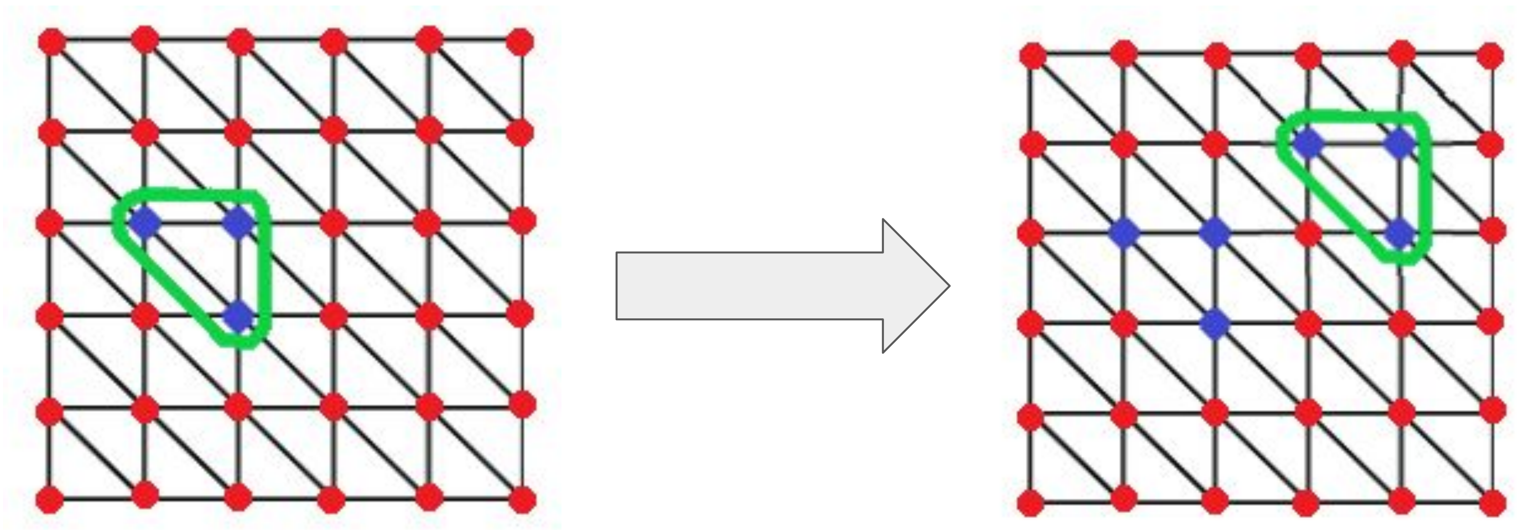
<u>Important Concepts</u>

Ergodic: Simulation will reach all possible states given enough time- a subset of the states can be used to represent the whole system

Detailed Balance: Transition probabilities are the same in both directions



- On each step, pick a random upside down triangle
- If that triangle is flippable (all three spins are oriented the same direction), flip those spins and save the configuration



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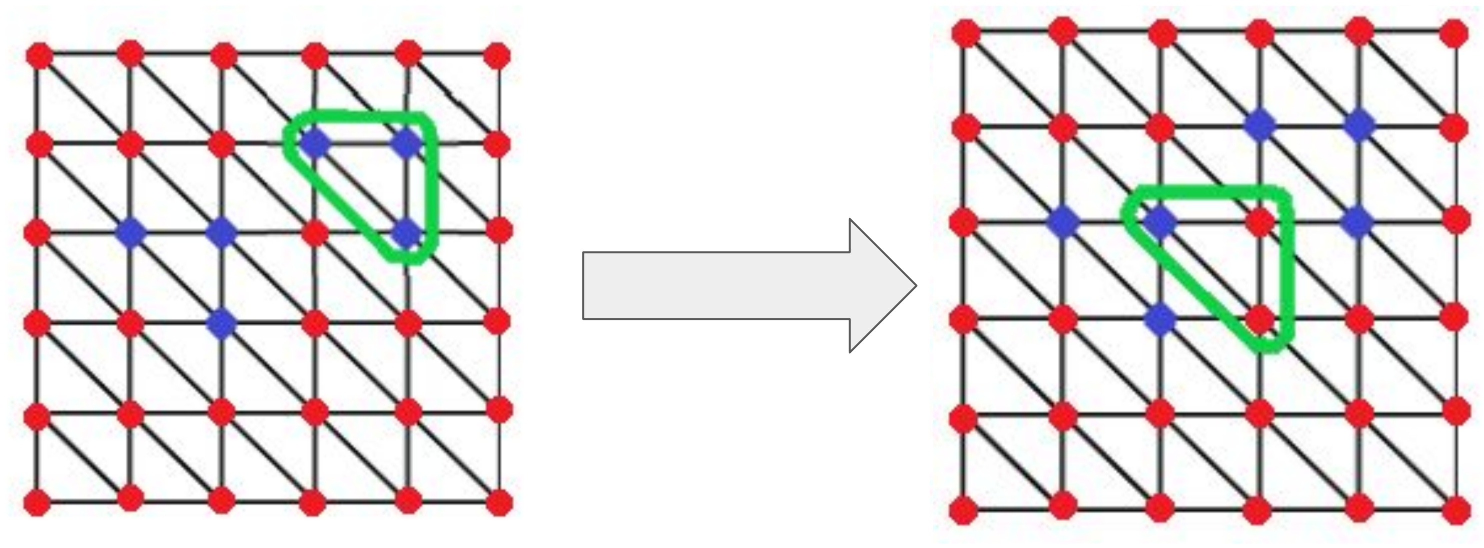
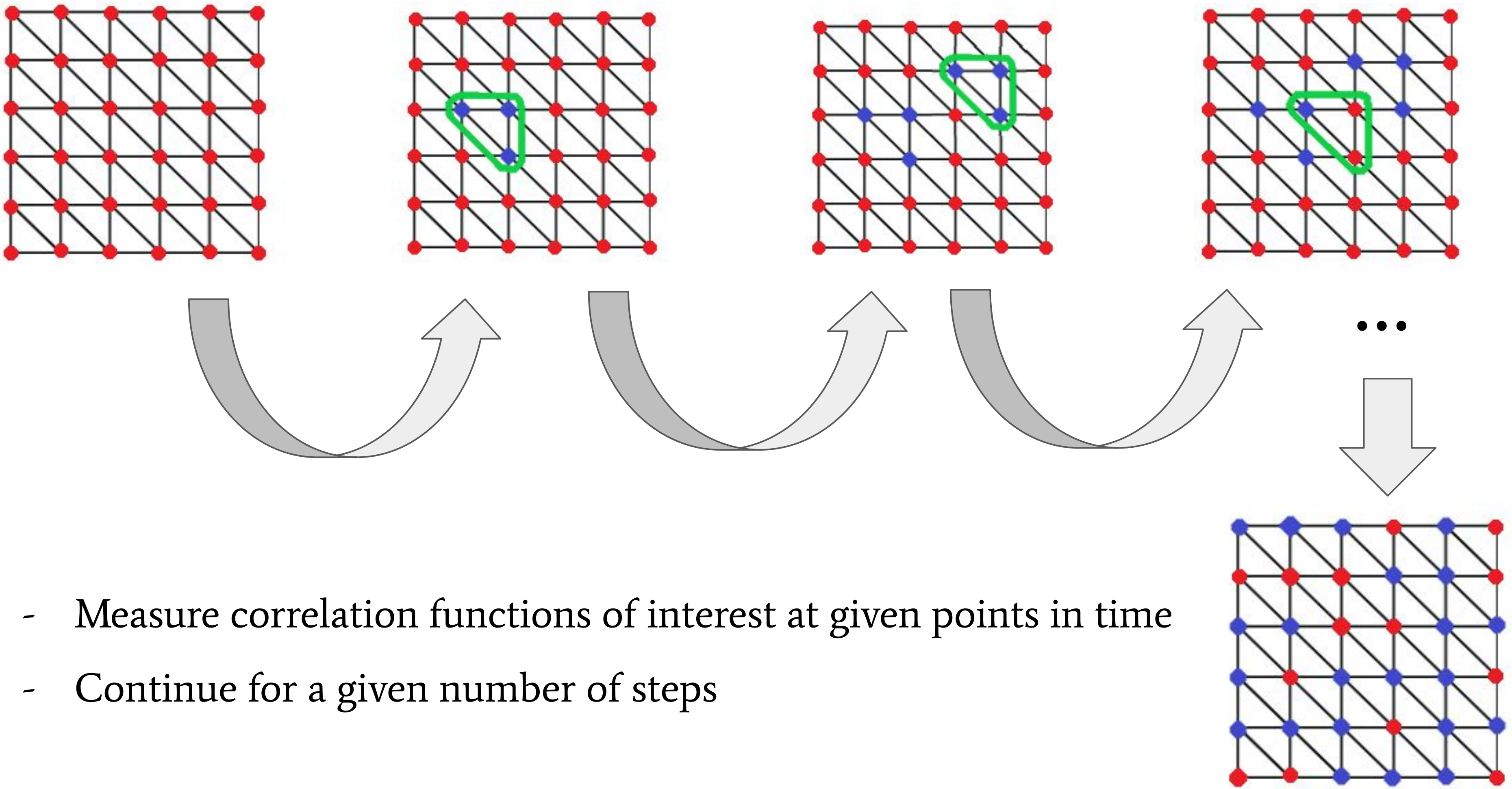
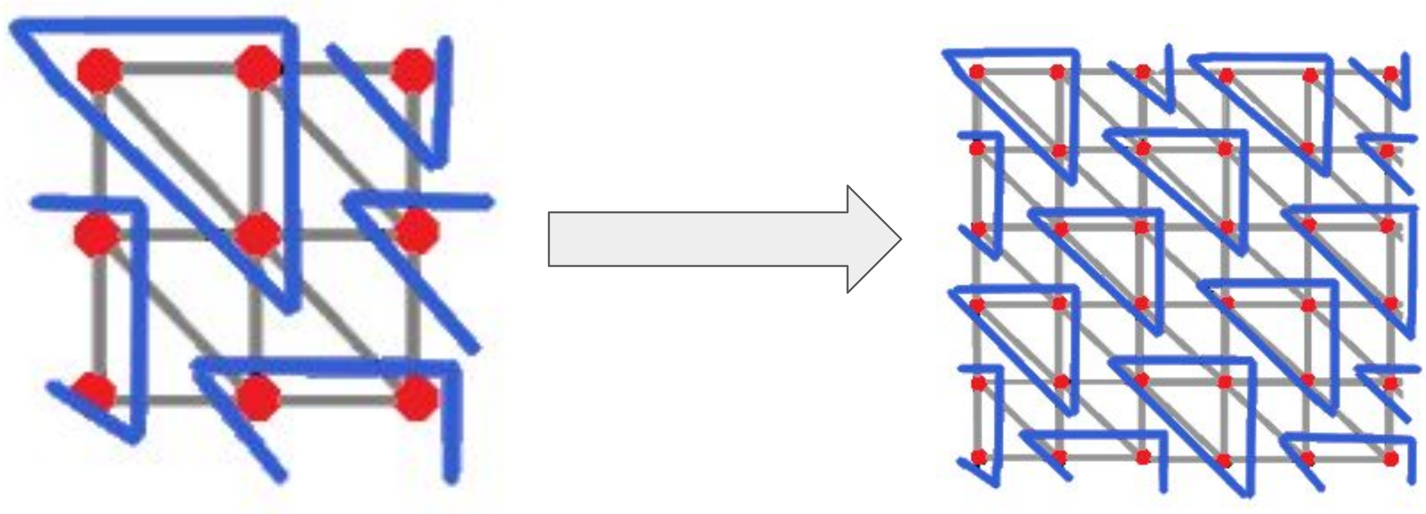


- If the triangle is not flippable, don’t flip and continue



- Measure correlation functions of interest at given points in time
- Continue for a given number of steps

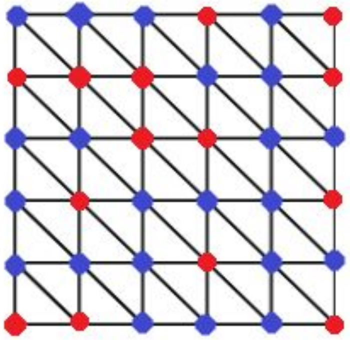
3×3 Trick for Global Update

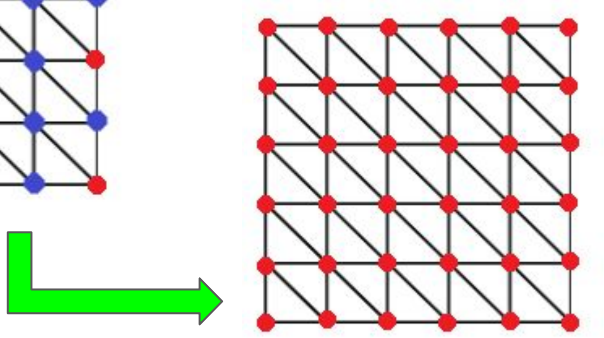


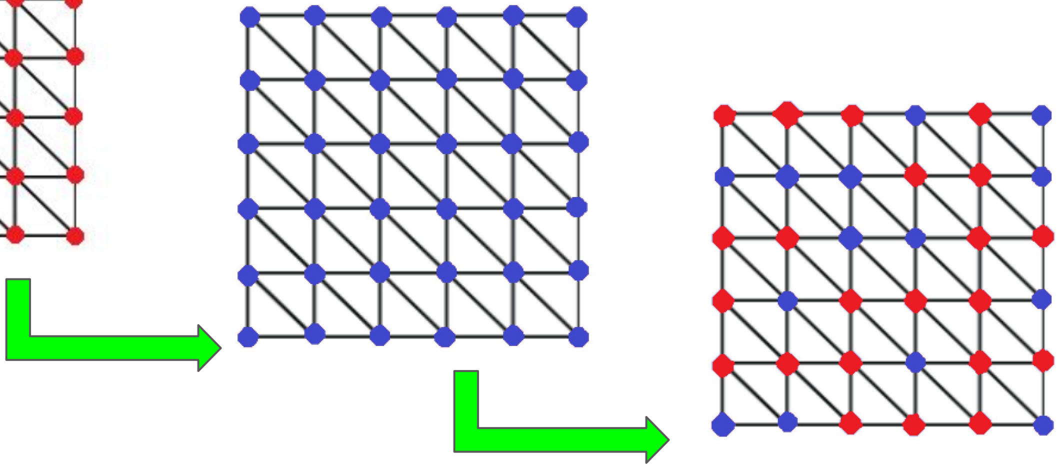
Exact downward triangle tiling on a lattice with side lengths of multiple 3

3×3 Trick for Global Update

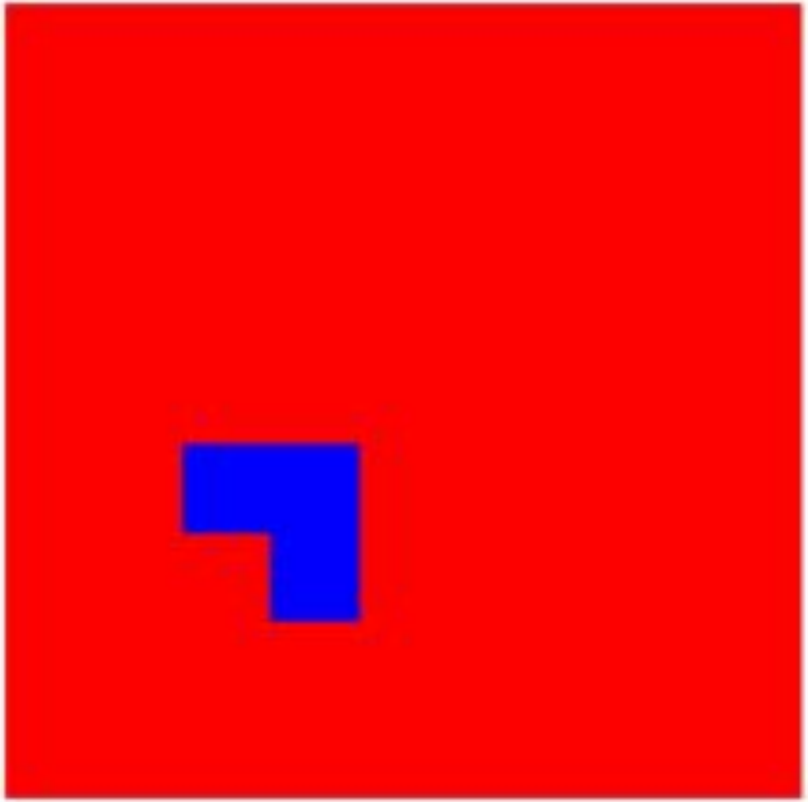
From this fact, it follows that every configuration can be completely flipped for a $3k \times 3k$ lattice.



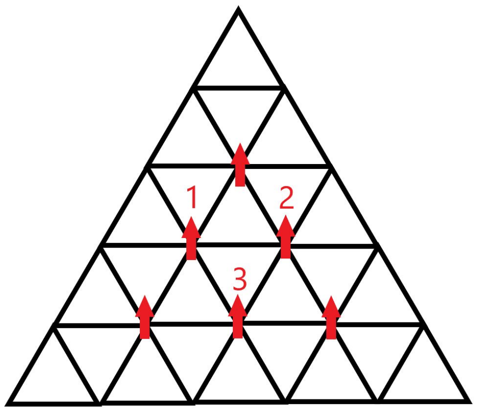




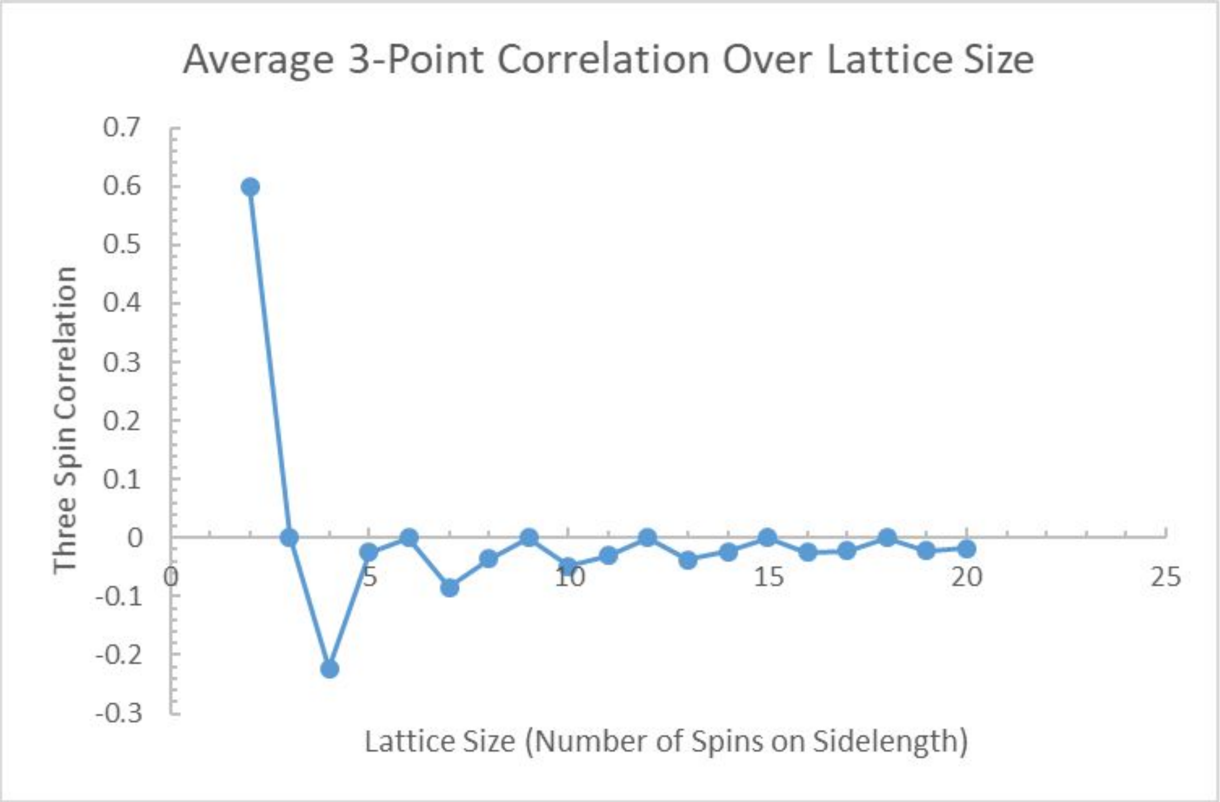
*Corollary: Three-spin correlations go to zero on these lattices



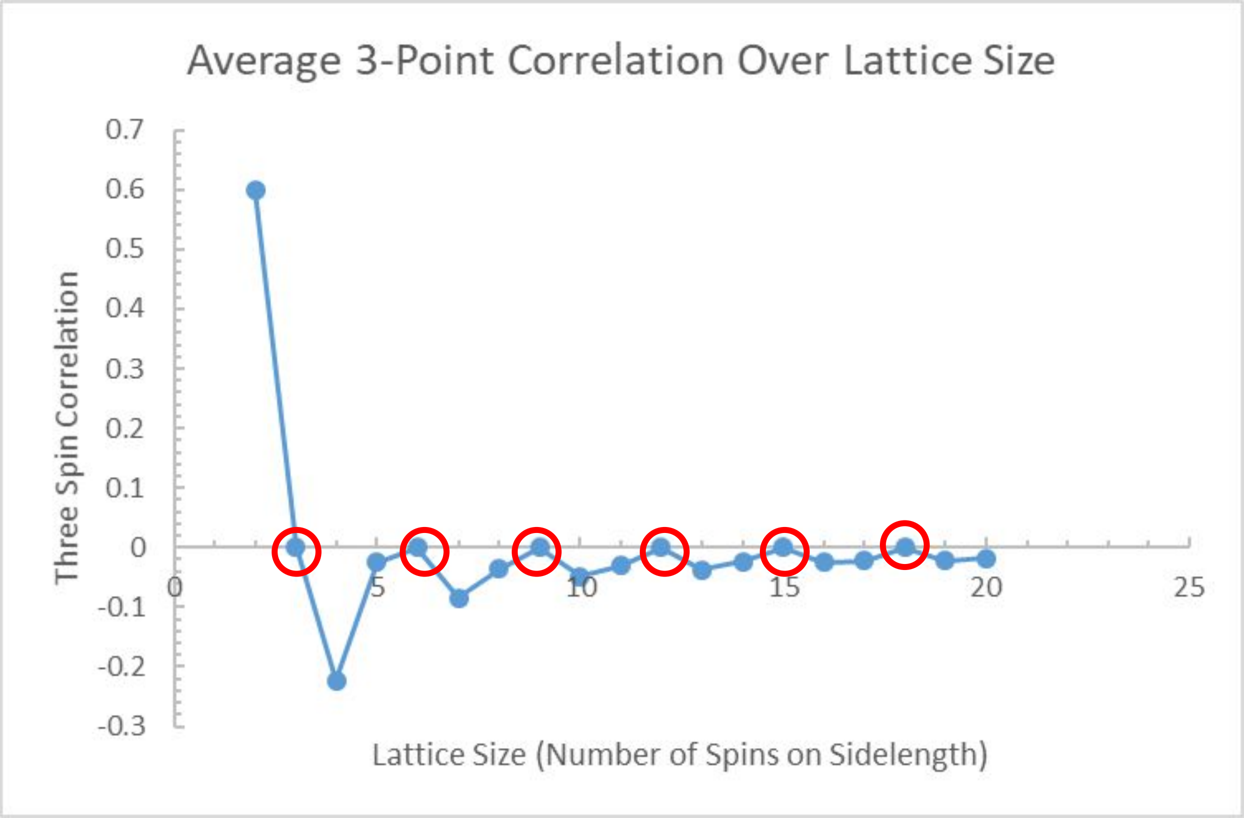
3 Point Correlation Function Results



$$\langle \sigma_1 \sigma_2 \sigma_3 \rangle$$



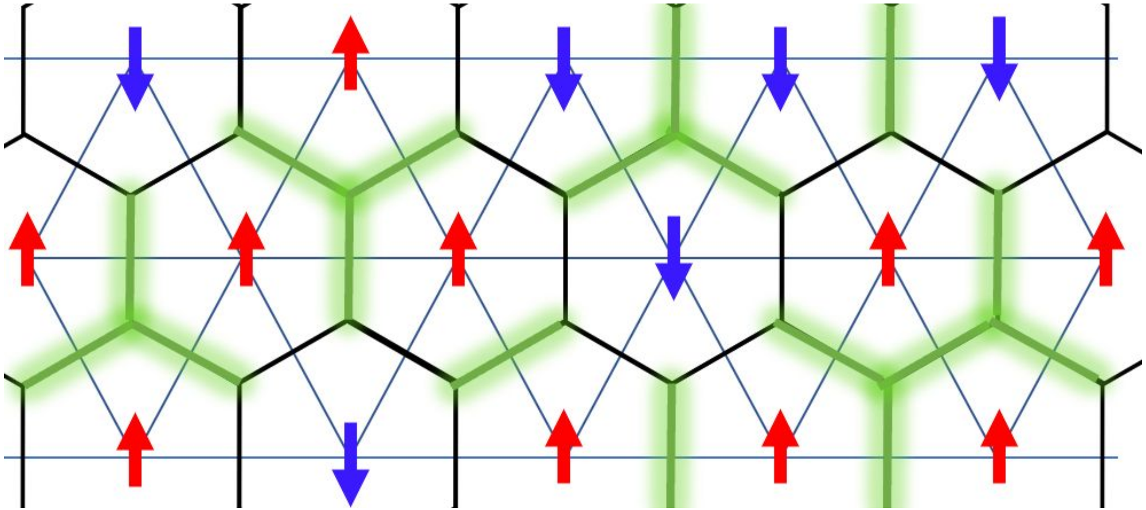
3 Point Correlation Function Results



Dimer-Dimer Correlations

There exists a hexagonal dual lattice to the triangle lattice

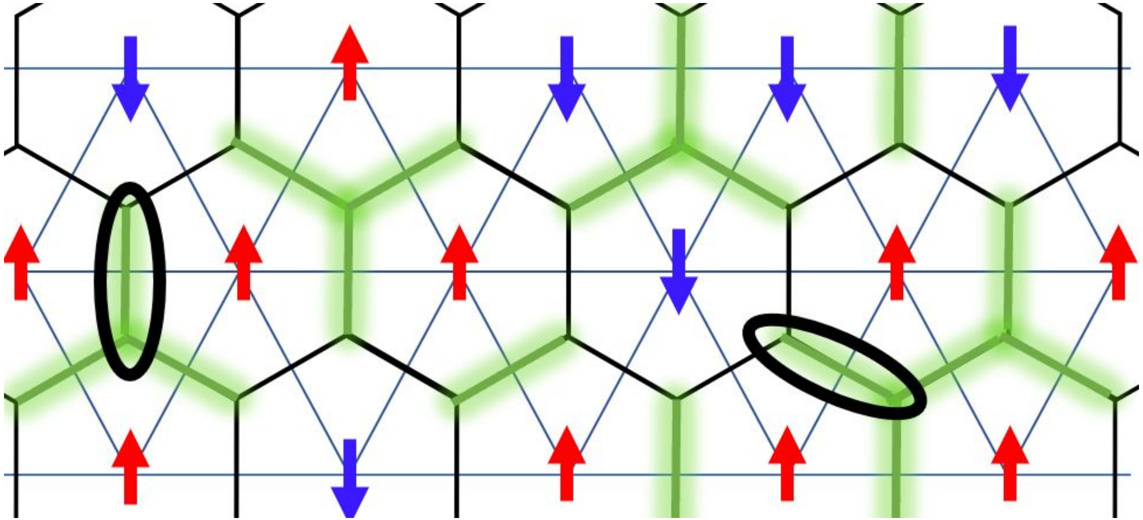
Links on the lattice correspond to bordering particles of the same spin- called 'dimers'

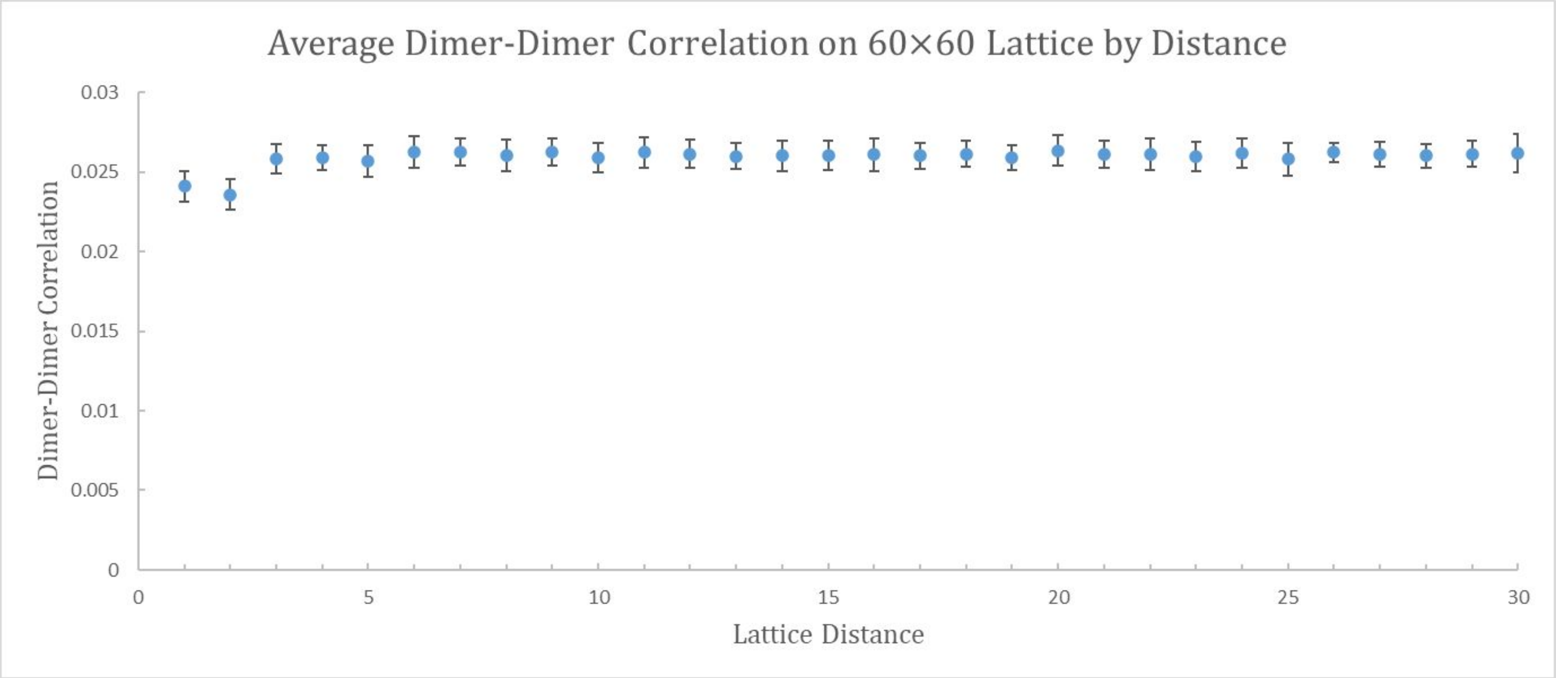


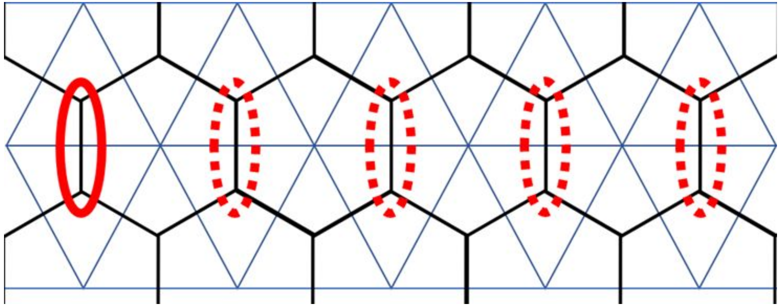
Dimer-Dimer Correlations

We can measure the correlations between two dimers over many configurations using the Monte Carlo simulation

Measures correlation involving 4 spins

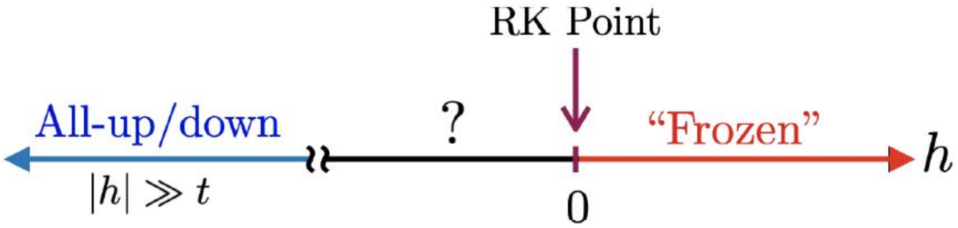



Average Dimer-Dimer Correlation on 60×60 Lattice by Distance
Dimer-Dimer Correlation
Lattice Distance
0.03
0.025
0.02
0.015
0.01
0.005
0
0
5
10
15
20
25
30

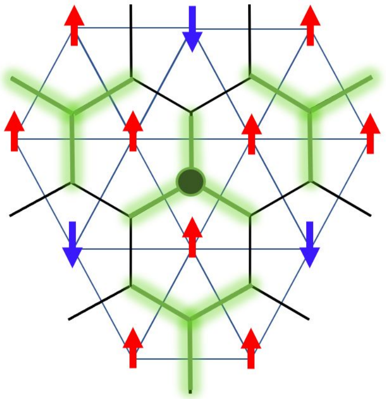


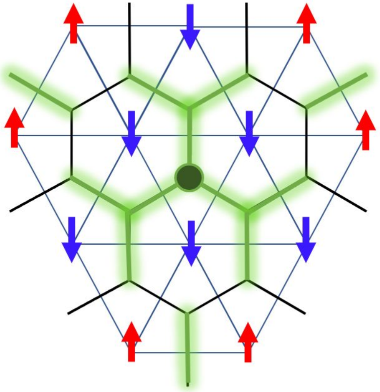
Future Investigations

- Flippable triangles as particles
- What are the dynamics of the flippable triangles?
- Ground states beyond the RK point



Graphic by Sagar Vijay (2022)





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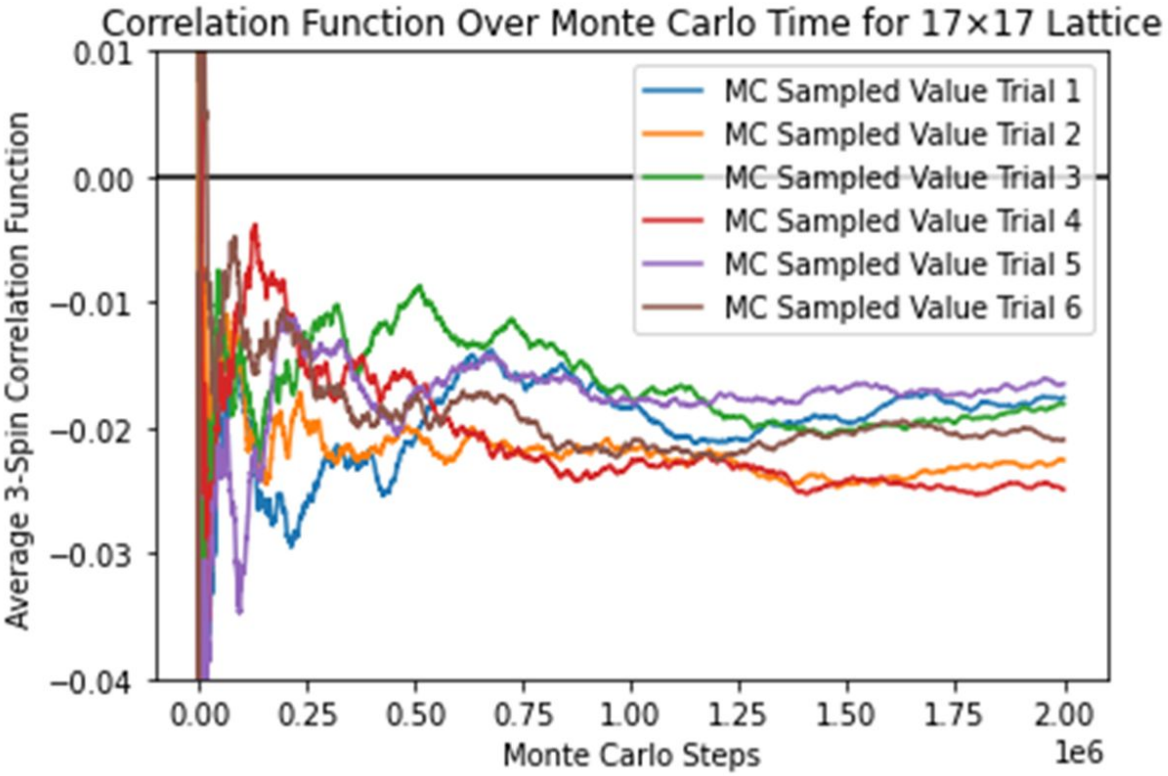
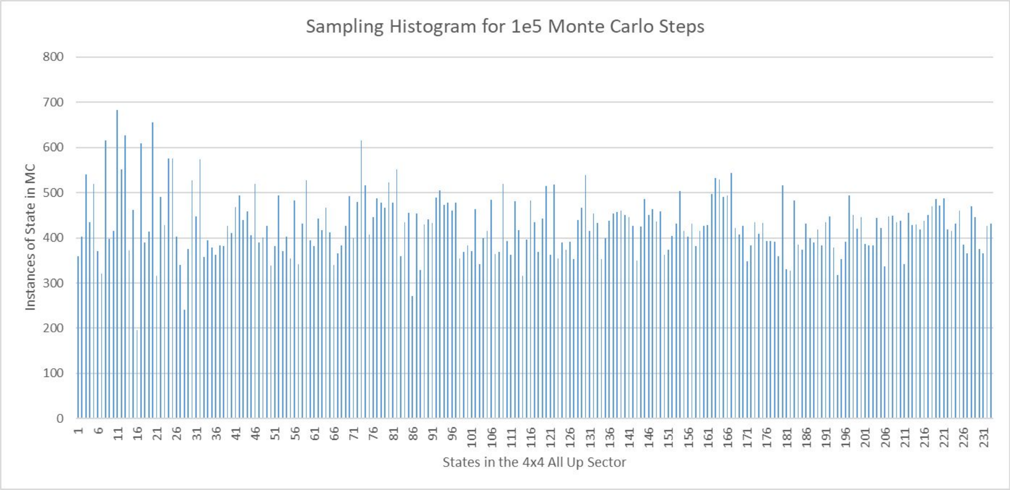
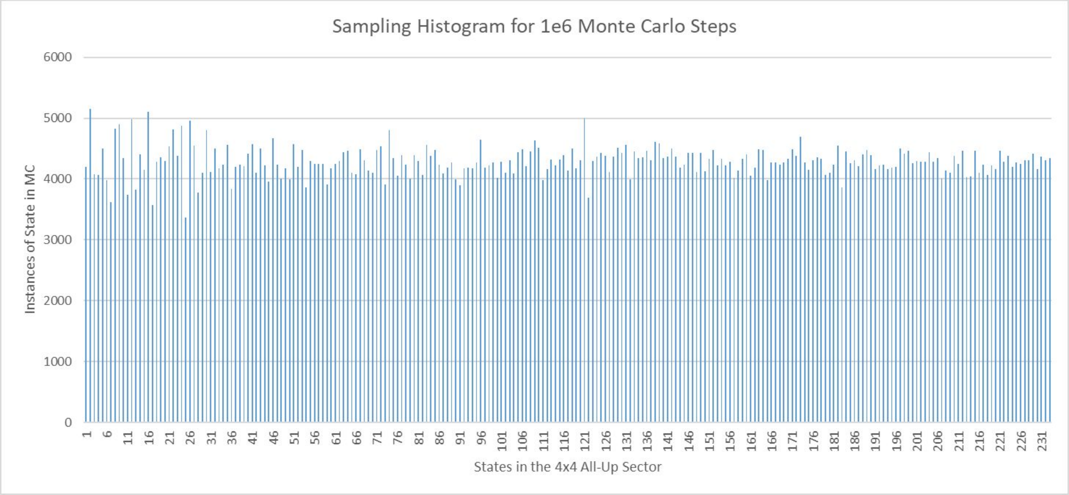
Further Slides are Additional Material- not part of main presentation


Correlation Function Over Monte Carlo Time for 17×17 Lattice
0.01
0.00
−0.01
−0.02
−0.03
−0.04
Average 3-Spin Correlation Function
MC Sampled Value Trial 1
MC Sampled Value Trial 2
MC Sampled Value Trial 3
MC Sampled Value Trial 4
MC Sampled Value Trial 5
MC Sampled Value Trial 6
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
1e6
Monte Carlo Steps



Sampling Histogram for 1e5 Monte Carlo Steps
Instances of State in MC
States in the 4x4 All Up Sector



Sampling Histogram for 1e6 Monte Carlo Steps
Instances of State in MC
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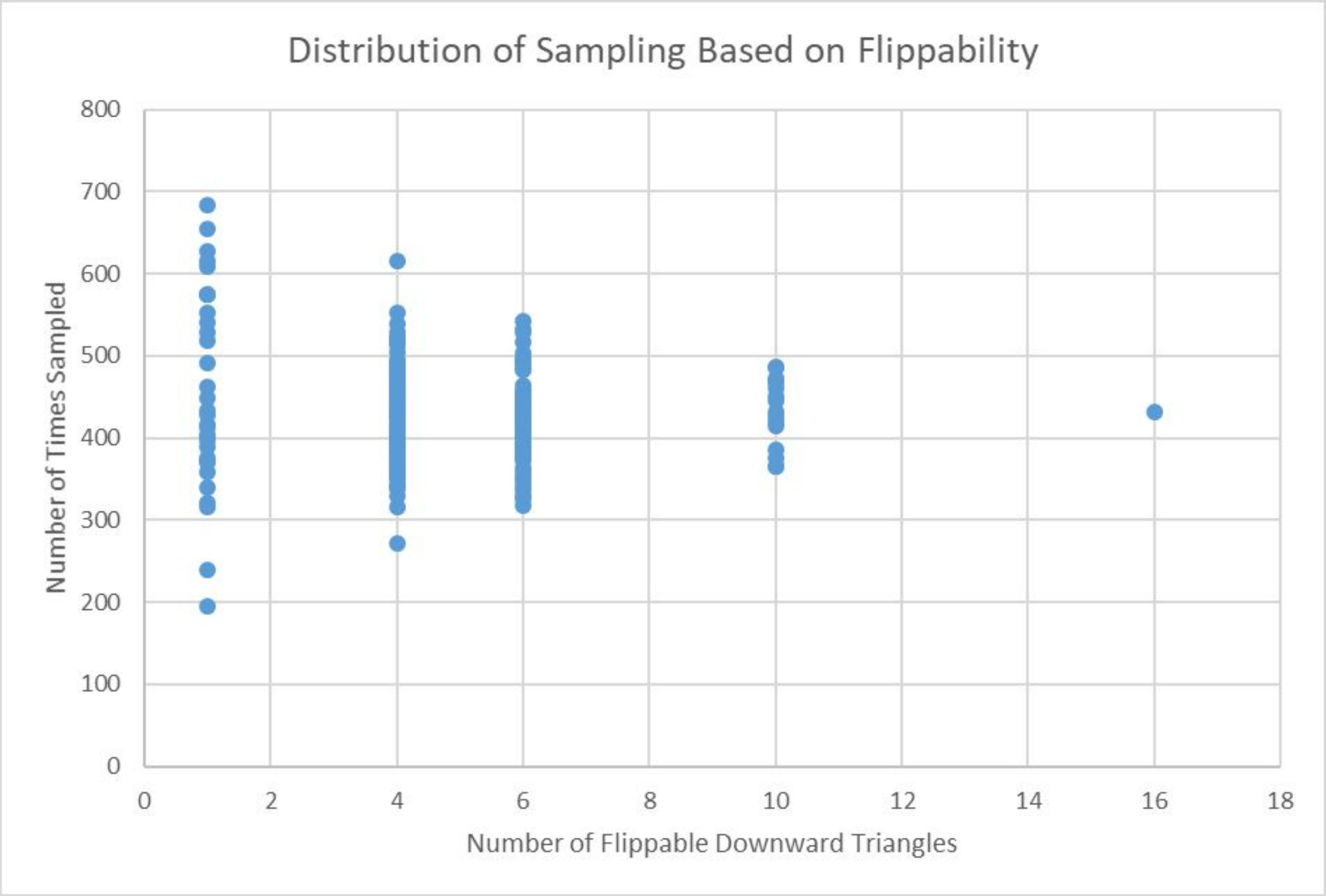

Distribution of Sampling Based on Flippability
Number of Times Sampled
800
700
600
500
400
300
200
100
0
0
2
4
6
8
10
12
14
16
18
Number of Flippable Downward Triangles