Exploring Chaos Through the Manhattan Lattice

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Quantum chaos has been an intriguing topic for the greater part of the last hundred years, but we lacked the technology and means to explore it further. The creation of Bose-Einstein condensates in ultra cold atomic gases in 1995 has given rise to direct and flexible probes of quantum phenomena at easily measurable scales of length and time. Using this principle, we investigate the potential for direct observation of chaotic dynamics in quantum systems. We use a numerical simulation to explore how this chaos presents itself under variable conditions.

I. INTRODUCTION

A. Chaos

In our world chaos appears in a wide variety of situations. Noticeable examples include: weather patterns, fluid turbulence, pinball games, and many more. Although there are so many expressions of chaotic behavior in our everyday lives, there is no rigorous definition of what chaos is. In this paper it will be referred to as a system where small changes, or perturbations, have an immense effect on its outcome. The chaos we are familiar with is called classical chaos and while we have quite a bit of knowledge on the topic, its counterpart, quantum chaos, is relatively under explored. We have created a quantum analog to a classically chaotic system, the Galton Board (1), to probe how quantum chaos strays from a classical system.

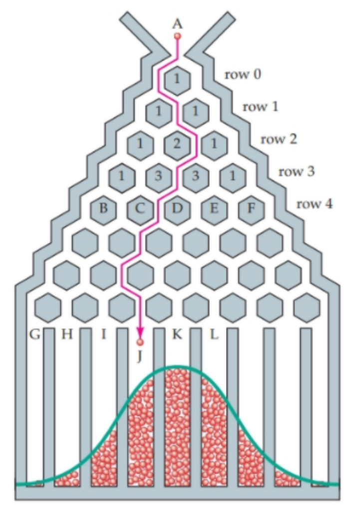

FIG. 1. This is a depiction of a classically chaotic system, the Galton board. It is a board covered in pegs with a spot at the top for a ball to be dropped. The ball then takes a random walk to the bottom where it lands in one of the spaces. [3]

1. *Classical*

Classical chaos is the nature of our world at a macroscopic level where small changes can have greater opportunity to create a "butterfly effect" and cause an outcome significantly different from its initial state. In many of these classically chaotic systems it is impossible to determine all of the conditions that influence a particular outcome. However, with precise trajectories and accurate measurements one could hope to accurately predict the outcome within some margin of error. With so many methods and techniques to analyze a classically chaotic system the goal of our research was to try to find a connection between classical and quantum chaos.

2. *Quantum*

The correspondence principle tells us that as a system becomes more macroscopic quantum phenomena and properties should approach the behavior of a classical system. This should mean that we can expect the same for chaos as well. Classical chaos has been described as the immense changes that result from the trajectories in a system, but quantum chaos is not so easily defined. When considering a quantum chaotic system each particle has a multitude of possible trajectories which makes the term "trajectory" ill-fitting in this sense. So in order to consider a definition that doesn't stray too far from the classical one we can describe quantum chaos in terms of the phase space or energy space [2].

B. Bose-Einstein Condensates

In order to create our "quantum Galton board" we need a ball to propagate through the board. Which is where Bose-Einstein Condensates (BECs) become extremely useful. A BEC is a phase of matter in which every atom has the same quantum state. By using a large number of particles it becomes much easier to experimentally observe quantum concepts at a macroscopic level. After cooling a gas to within a few nK of absolute zero it becomes a quantum gas. At which point the particles in the cloud are all in a minimum velocity state. We can see this in Figure 2 with the velocity distribution of a rubidium BEC.

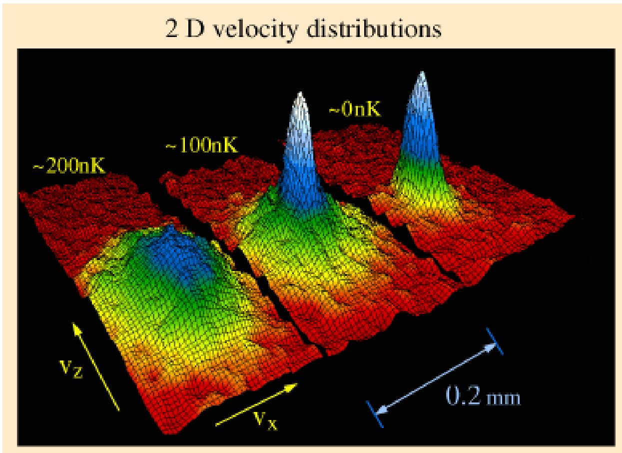

FIG. 2. This is a graphical representation of a rubidium BEC. The velocity distribution shows that the colder you get the atoms in the cloud the closer they get to their minimum velocity state. At their minimum velocity the particles all want to be in the ground state which creates the Bose-Einstein Condensate. Every atom in the BEC cloud is in the ground quantum state and therefore also each has the same wave function describing their existence. [1]

Another important reason to use a Bose-Einstein Condensate for this project is because we are trying to observe the time evolution of the cloud's wave function. A large issue arises in quantum mechanics when a wave function is observed: it collapses to a single probability. However, with a BEC this is not such a detrimental flaw to the experiment due to the large number of particles in the cloud [1]. As they collapse they will statistically distribute themselves according to their common wave function and allow us to get a representation of an unobserved particle evolving in time.

C. The Manhattan Lattice

Arguably the most important step in creating our version of the Galton board is the peg-board on which the system propagates. An optical lattice is used in our simulation and experimental setup. This optical lattice has been aptly named the Manhattan Lattice (Figure 3) for its "streets" of low constant potential energy and its "skyscrapers" of high potential energy.

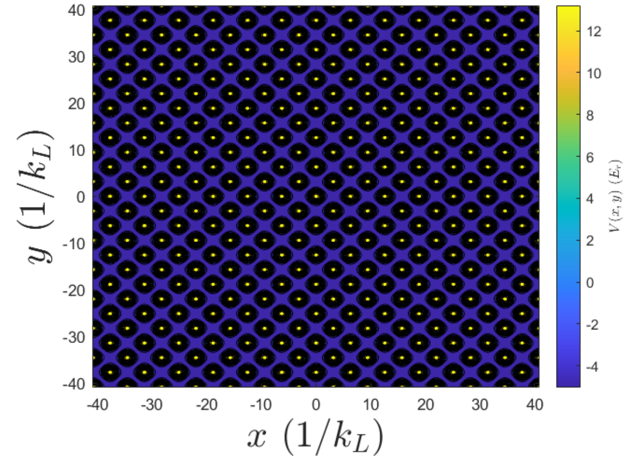

FIG. 3. The Manhattan Lattice: Grid of standing waves of light, an optical lattice, where a BEC is constrained and then allowed to scatter.

The potential for the lattice can be described in 2-D by the following:

$$\bar{V}(\bar{x},\bar{y}) = \frac{1}{2}V_X\cos(2k_L\bar{x}) + \frac{1}{2}V_Y\cos(2k_L\bar{y}) + 2\sqrt{V_XV_Y}\cos(2k_L\bar{x})\cos(2k_L\bar{y})$$

Where V_X and V_Y are each the potential energy depths of the lattice in each direction, k_L is the wave number of the lasers in the beams, and the bars represent dimensionalized values. With beams of equal power in each direction the equation becomes [5]:

$$\bar{V}(\bar{x},\bar{y}) = \frac{1}{2}V_0\cos(2k_L\bar{x}) + \frac{1}{2}V_0\cos(2k_L\bar{y}) + 2V_0\cos(2k_L\bar{x})\cos(2k_L\bar{y}) \quad (1)$$

With a rigorous definition for the potential energy in the lattice beams we can discuss the experimental and theoretical methods driving the simulations and results of our research.

II. METHODS

Although there is a large divide in physics research between experiment and theory, in order to progress in the name of science we must acknowledge the usefulness and contributions of both. This project involves heavy use of both aspects to function and thus will be outlined in this section. Both of the following two sections were the driving force behind the simulation program with which I conducted my research.

[1] $8 \times 10^4 \pm 10\%$

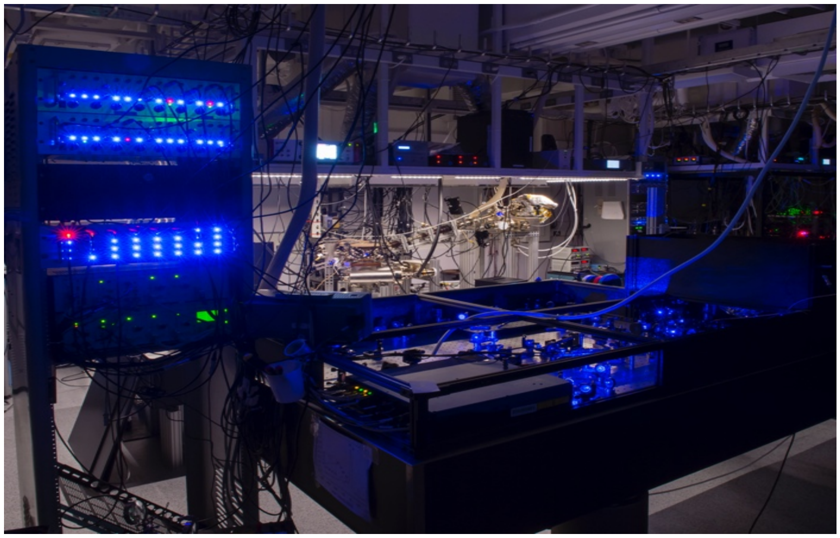

FIG. 4. An image of the Strontium Machine at Weld Lab, UCSB. [6]

A. Experiment

The experiment in a lab environment involves creating a BEC and then imaging the wave function as it scatters in the Manhattan lattice. To create the BEC in Weld Lab at UCSB a cloud of atoms are optically cooled to near absolute zero. The cloud becomes a BEC after reaching the condensation temperature described by the atoms' mass and the cloud density. At this point the cloud is cold enough to condense into a common ground state. This process takes place throughout a number of steps within the Strontium Machine.

Experimentally, creating the BEC is only the first step for this system. In the near future Peter Dotti at Weld Lab hopes to use lasers to realize a lattice[2] of this nature. Then through the use of the Strontium Machine to create a ^{84}Sr BEC the chaotic dynamics of the system can be imaged as the cloud scatters throughout the lattice.

III. SIMULATION

The simulation process involves creating a BEC cloud in an initial state within the Manhattan lattice and then allowing its wave function to evolve over time. In order to keep our procedure similar to that of the Galton board the simulation also involves a ramp phase in which the energy of the system is decreased, otherwise our Galton board comparison is meaningless and we are no longer examining the physics of quantum chaos.

Beginning with the wave function consistent with the optical dipole trap (ODT) potential we can use a harmonic oscillator to model the system numerically. While ignoring particle interactions we find that the BEC after condensation is a ground state harmonic oscillator and can be used for our initial state. With a result of solving the Schrödinger Equation at time and position 0 we can finally begin the time evolution of our system.

A. Ramp and Free Evolution

The next step in the process is to ramp the potential energy of the lattice slowly over a short period of time. This portion of the procedure is essential to observing the dynamics of the system. Of course we could start with an initial wave function, release the optical dipole trap, and observe its propagation, however, this is contradictory to our goal of examining a low energy chaotic structure. In addition, at such high energies it becomes much more difficult to effectively record useful data when the particles scatter so quickly. The solution then is to make sure that the cloud can fully interact with the lattice and be imaged properly. This is the ramp phase mentioned previously. By slowly increasing the potential of the lattice underneath the BEC for around 80-100 ms, or until V_{lat} approaches that of the ODT, we can explore the chaos of a low energy system.

Once the ramp period is finished, the wave function has its trap removed and the particles are then allowed to scatter in its free evolution phase. This segment of the simulation is imperative to researching the dynamics because it is during this frame where the interesting chaotic structures may appear. Some constraints should be kept in mind for the free evolution period however, namely, should the wave function encounter the edge of

[2] Although the Manhattan Lattice is referred to as such, it is not a lattice in the typical sense because it does not hold atoms in local minima. This terminology is used to draw a connection between the similar geometry of each.

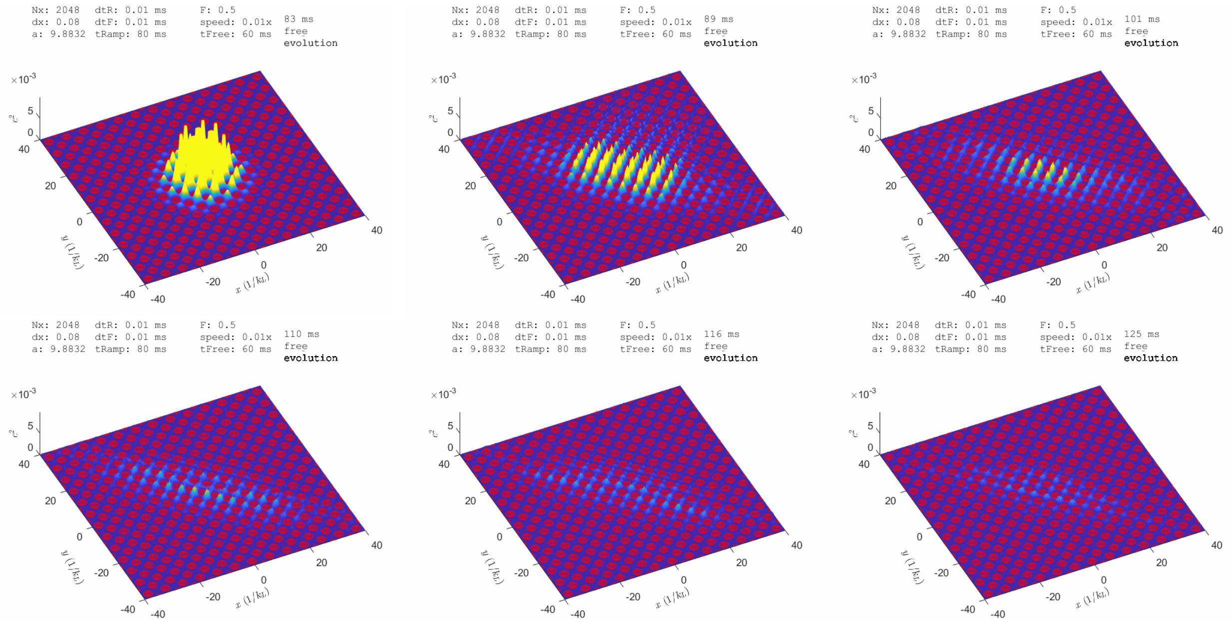

FIG. 5. This is an image made from 6 frames of a gif simulation with a large force applied at 45 degrees to the horizontal. You can see that the wave function exhibits high frequency Bloch oscillations.

the defined meshgrid issues arise[3].

IV. EXPLORING CHAOS

After the development of the simulation in Summer 2020 by Addison Hartman, the logical next step was to start searching for signatures of chaotic dynamics using the quantum platform that was created.

A. Applying a Force

In order to uphold our quantum Galton board analogy we began by applying an arbitrary force to the wave function as it evolves to see what effects are introduced. Beginning with a weak force I ran simulations using many different angles, oscillator lengths, and time steps. The goal was to observe interesting or unique dynamics in the lattice sites over the course of the simulations, however, simulating a small mesh size introduced some issues and prevented a realistic physics simulation environment. These issues will be described in greater detail later.

After testing extensively with weak forces we performed many of the same simulations with forces at different orders of magnitude. This change introduced some unexpected results which are outlined in §VA.

[3] See §IVB for more detail.

B. Boundary Effects

As stated previously, some issues were encountered when trying to simulate a smaller sized mesh of points, namely, these issues are boundary effects. These occur when the position-space evolution of the wave function encounters the edge of the defined meshgrid of points that the computer has generated. Thus, when the computer tries to propagate the next step in time it misinterprets the data and causes the wave function to appear as though it is "reflecting" backwards, similar to a wave in a pool hitting a wall. Other boundary effects include the spikes appearing on opposite walls and generally introducing dynamics that are nonphysical. These nonphysical results threw a wrench in our progress and prevented any real results to be uncovered for quite some time.

One way to mitigate these troubles is to end the simulation before the wave function has the chance to encounter these effects. However, this solution limits the ability of our program to simulate larger systems with dynamics that take longer to introduce themselves. Which left us with one option: simulate a larger mesh size. Although this solution removes the physical issues of the simulations, using a significantly larger mesh requires enormous amounts of computing power and can easily overflow a personal computer's RAM capabilities.

1. *The Cluster*

In order to create these large simulations without putting a massive toll on the hardware of my own laptop, we decided to move our code onto the UCSB Center for Scientific Computing computer cluster. With hundreds of cores and leagues more RAM we could run a significantly larger mesh at no cost to my own resources. After finally eliminating any withstanding problems with our program we were able to begin looking deeper into the dynamics of these chaotic systems and find some results.

V. RESULTS

Exploring the many different parameters provided us with the data to draw some conclusions, but also left a few new questions in its wake. The tests that were run with differing angles of force being applied showed no significant or interesting behavior as one might expect from the analogy created by our quantum Galton board. We had anticipated the emergence of chaotic behavior similar to that of classical chaos, however, the difference in force angle seemed to have little to no profound effect. In contrast, the force magnitude seemed to produce the most surprising results. There is a large disparity in the dynamics when strong and weak forces are applied. We noticed that for low force values (on the 10^{-3} or smaller magnitude) the particles in the BEC behaved ballistically as we would expect. They moved in a manner that was indicative of the direction of the force being applied. However, for larger forces we saw the wave function behaving as Bloch oscillations.

A. Bloch Oscillations

Bloch oscillations in position space were first physically observed by the members of the Weld Lab at UCSB, and first predicted in 1929 by Felix Bloch. This phenomena occurs when particles under a constant force are also being affected by a periodic potential [4]. The combination of the applied force and potential lattice cause the particles' momentum to also behave periodically. We saw these oscillations in our simulations when a strong force is applied, and, proportionately, stronger forces applied yields a higher frequency of oscillation. In figure 5 you can see these high frequency oscillations in the wave function. In the future it will be important to look further into the appearance of these Bloch oscillations without necessarily trying to simulate them, as well as the differences in strong and weak forces and how they induce this type of behavior.

B. Incommensurate Mesh

Another important discovery made during the course of this project was the importance of having a commensurate mesh, or that the meshgrid points and lattice minima occur at rational multiples. Since the Manhattan Lattice is not a typical lattice and particles are not held in these minima, it becomes ever more important for the "streets" to be constant low potential to observe the dynamics we are looking for. However, we noticed that due to the irrationality of $\sqrt{2}$, the angle that the meshgrid is created at compared to the lattice causes the simulation to behave in an undesired manner. Thus, the logical solution was to rotate the Manhattan Lattice by 45 degrees so that the irrationality is no longer an issue. We are not completely certain whether or not this problem could invalidate our previous data so extensive re-testing of some of the more interesting simulations is necessary to fully explore this chaotic system. Although after changing to a commensurate mesh I re-evaluated the force being applied at different angles, yet we saw nothing new to report on that avenue. Therefore we can conclude that varying the angle of the applied force yields little to no effect on the chaotic outcome, however, the magnitude has a significant bearing on the wave function's behavior.

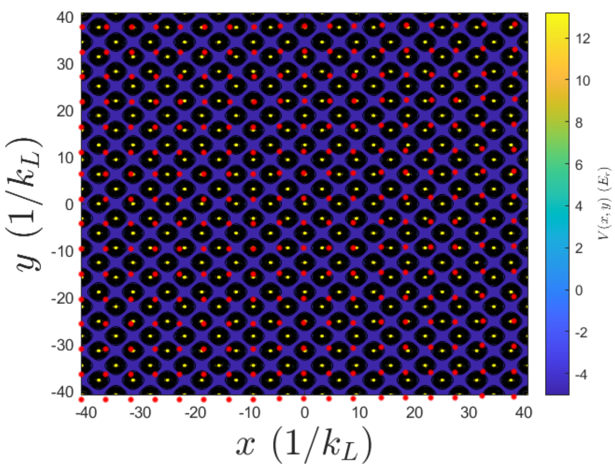

FIG. 6. Image created to show how having irrationally space lattice sites causes the rationally spaced meshgrid points to occur in an incommensurate nature.

VI. FURTHER WORK

The term length for the REU program at UCSB has ended, howver, there is still a great deal of work to be done on this project. The simulations are still yet to be perfected and need to be looked at more carefully in a physical sense. One way to further improve upon the computations being done would be to add particle interactions. The velocity of ultracold atoms is so small that these interactions may be few and far between, but including them may give rise to some yet to be seen chaotic behavior within the BEC. To do so, solving the Gross-Pitaevskii equation, or GPE, as opposed to the Schrödinger equation as well as using a Thomas-Fermi distribution instead of a quantum harmonic oscillator are necessary.

Another step in the future is to use the commensurate mesh to look at some of the previously attempted simulations. It may be valuable to ensure that "no stone is left unturned," so to speak. Specifically, it would be interesting to vary the cloud size and look at the interference that the wave function may exhibit in the lattice sites. Potentially researching more into the causes behind the appearance of Bloch oscillations in our simulations is another lucrative avenue to pursue. Perhaps this dichotomy between strong and weak forces is an error in our system, or maybe it is an unforeseen effect of our solution and evolution of the wave function. Lastly, in the future a laboratory setup for this experiment is pertinent to look even deeper into the nature of a quantum chaotic system. Although there is bound to be many more challenges presented to this project, the field of quantum chaos is rich with discovery waiting to happen.

VII. ACKNOWLEDGEMENTS

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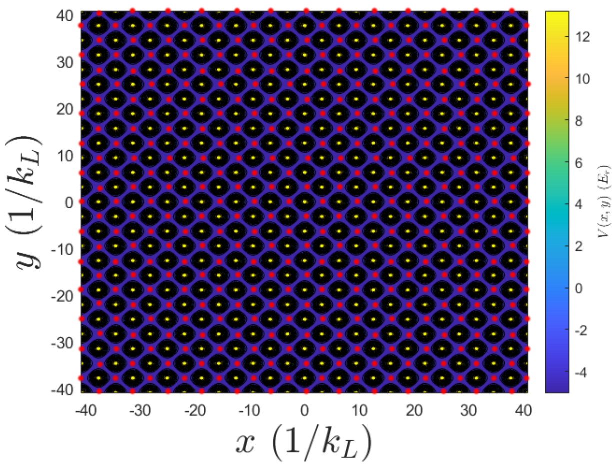

FIG. 7. Image created to show how rotating the lattice 45 degrees can remove the undesired effects of an incommensurate grid, thus creating a commensurate simulation.

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