



Using the Manhattan Lattice to Simulate Quantum Chaos

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Outline

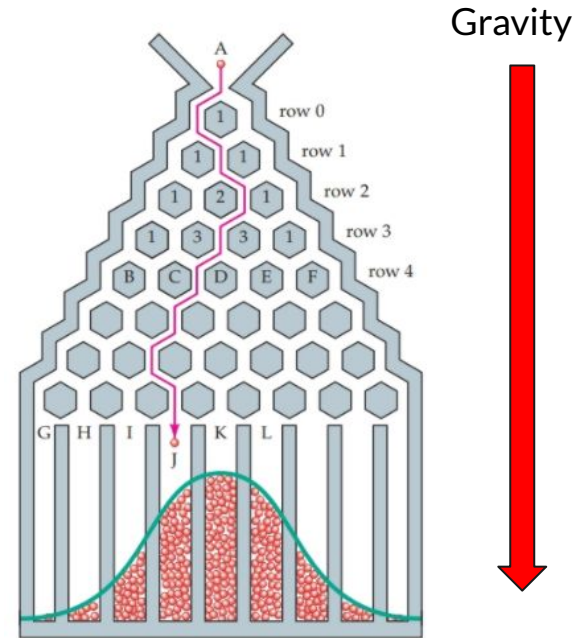


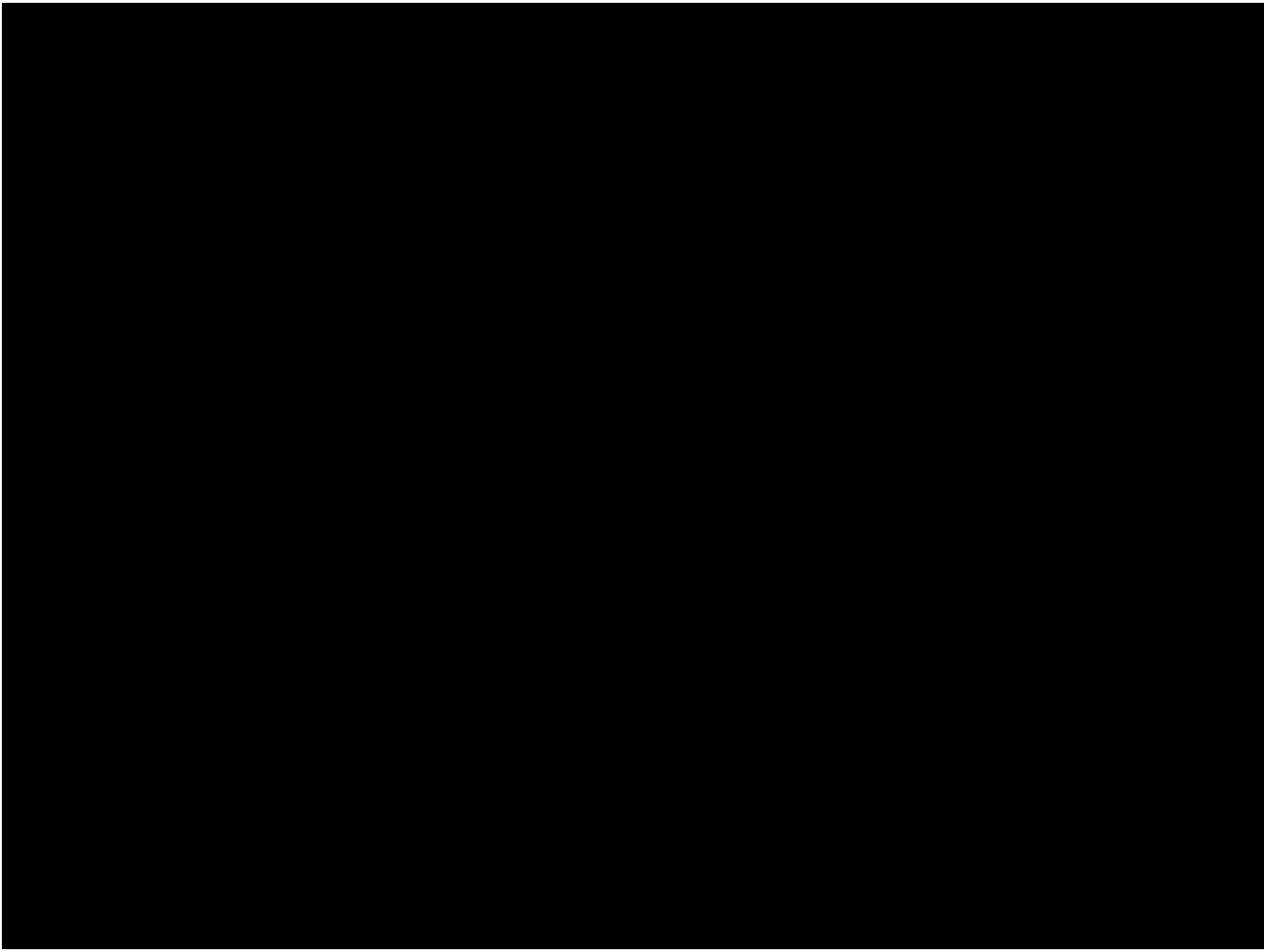
- ❖ Chaos
 - Classical and Quantum
- ❖ Bose-Einstein Condensates
- ❖ The Manhattan Lattice
- ❖ The Simulation
- ❖ Exploring Chaos
 - Bloch Oscillations
 - The Wrong Lattice?
- ❖ Conclusions/Future Work

What is Chaos?

A system where small perturbations have an immense effect on the system's dynamics

A classical example is of the Galton board (shown at right)







Chaos in Different Regimes

Classical:

- Described by trajectories
- Occurs often in macroscopic objects
- Extensively and actively explored

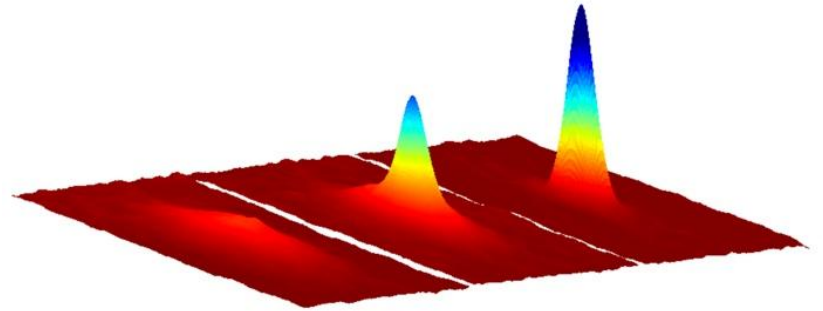
Quantum:

- More easily described in terms of the allowed energy levels rather than dynamics
- Uncommon in the world we interact with
- Under explored and rich with potential

A Bose-Einstein Condensate (BEC)

A phase of matter where every particle is in the same quantum state

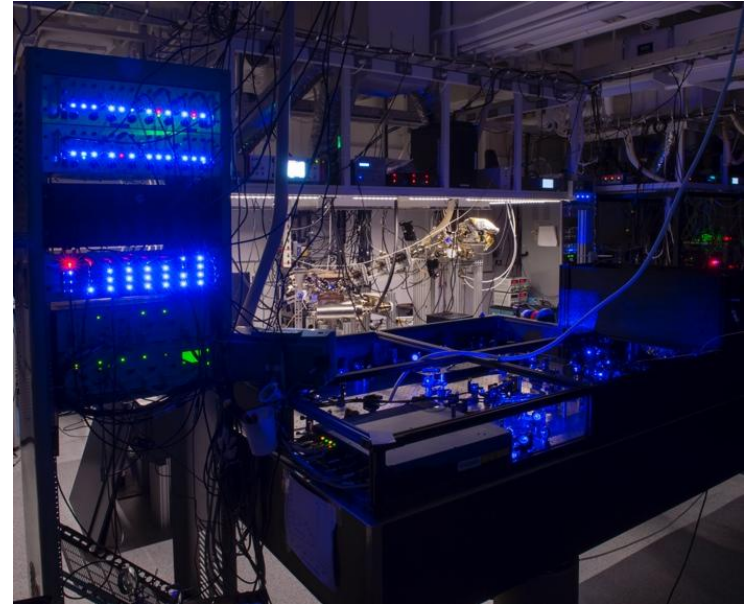
Can be used to examine quantum properties at a macroscopic level!



The Strontium Machine

At Weld Lab, it optically cools samples of ^{84}Sr to within 10nK of absolute zero

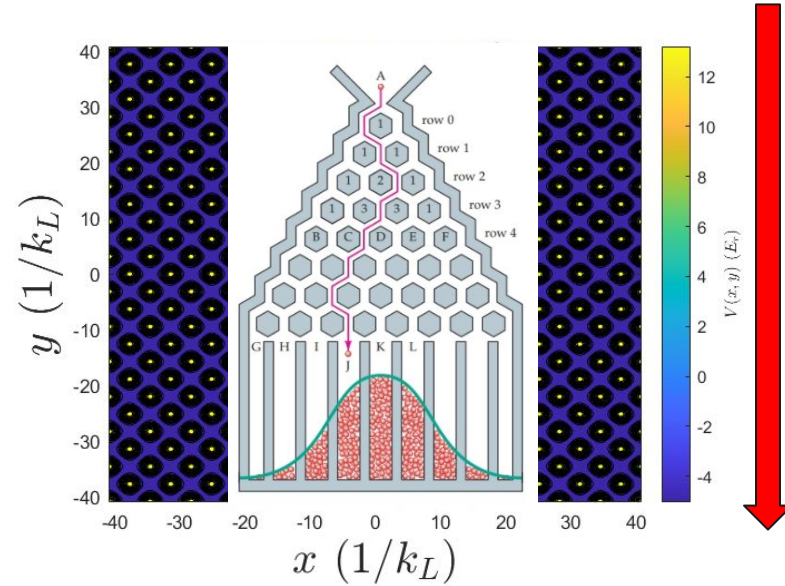
Involves multiple stages of laser cooling, optically trapping, and collimating atoms to create a BEC



The Manhattan Lattice

An optical lattice with “streets” of low constant potential and “skyscrapers” of high potential

Functionally serves as the pegs on the Galton board with the Bose-Einstein condensate as the ball





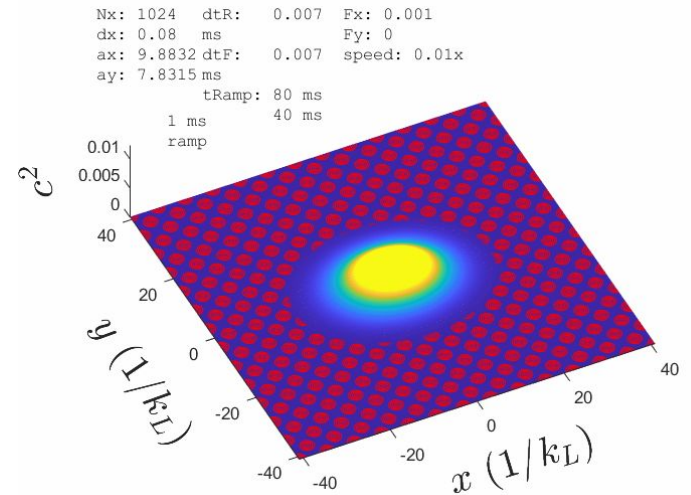
Observing Quantum Phenomena

Quantum mechanics dictates that measuring a system will cause the affected wave functions to collapse

Observing BEC wave function causes collapse and probabilistic distribution

The Simulation

- Initial state: the ground-state wave function at time and position zero
- Ramp phase: Used to slowly raise the potential energy of the lattice
- Free evolution phase

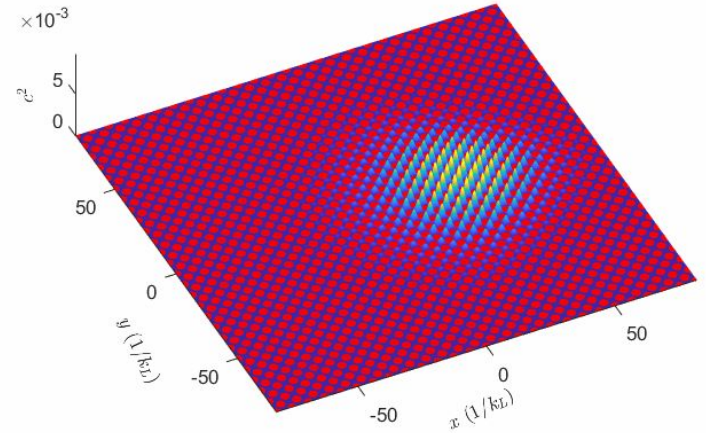


Searching for Chaos

Studying the effects of applying an arbitrary force during the evolution of the wave function

In hopes of noticing some signatures of chaotic behavior!

Nx: 2048 dtR: 0.01 ms F: 0.001
dx: 0.08 dtF: 0.01 ms speed: 0.01x
a: 9.8832 tRamp: 80 ms tFree: 20 ms

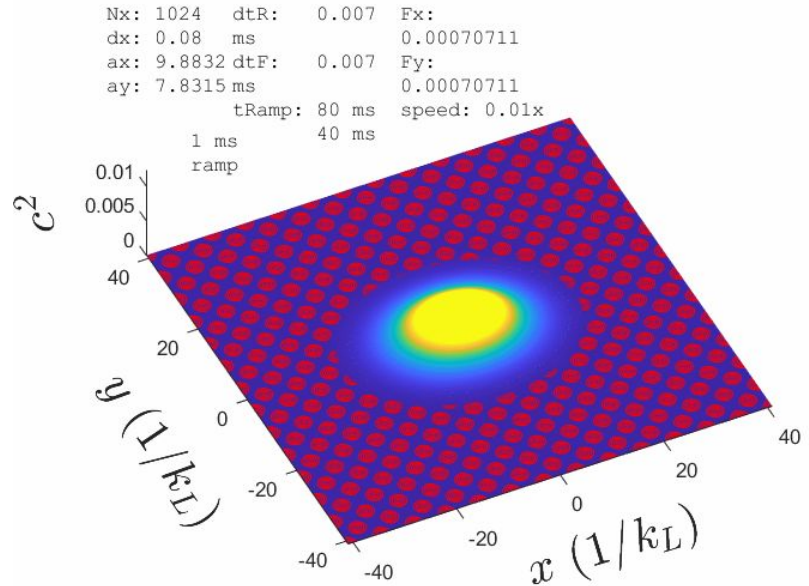


Trouble with Boundaries

A big issue with using a small number of points:

Encountering boundary effects

As the wave function reaches the edge of the axes, the program misinterprets the next step in its evolution and appears to “reflect” the distribution





The Cluster

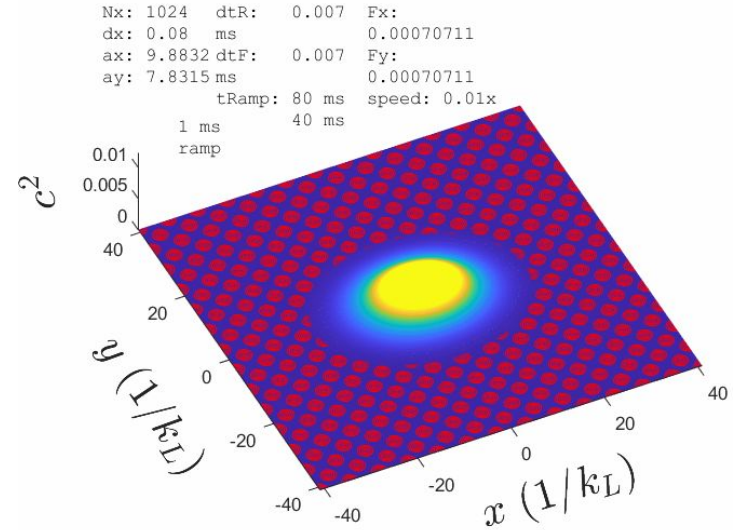
I was approved for an account on the Center for Scientific Computing cluster at UCSB

Using the parallelized threads we can run much larger simulations at a lower computational cost!



Strong vs. Weak Force

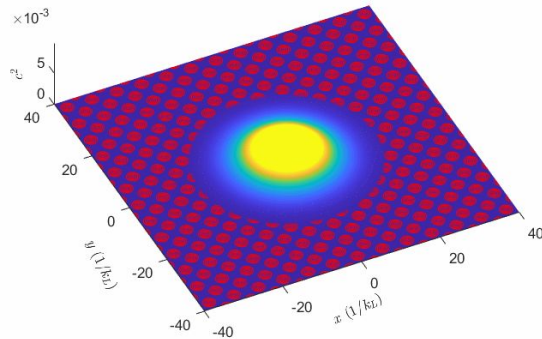
Applying a small force to the BEC cloud yields ballistic behavior, but a strong force causes the particles to show Bloch oscillations



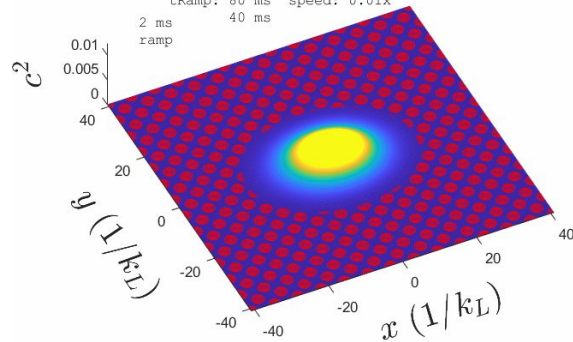
Bloch Oscillations

When a constant force is applied to a particle in a periodic potential the momentum also begins to behave periodically

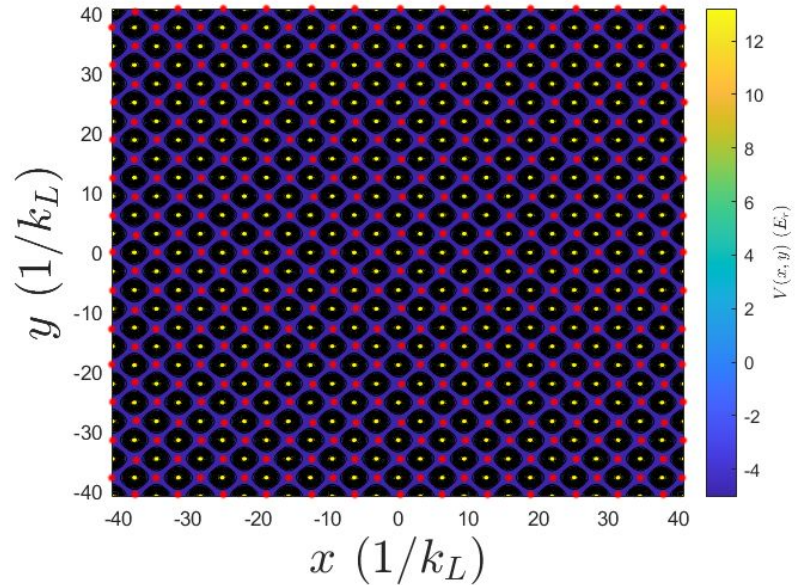
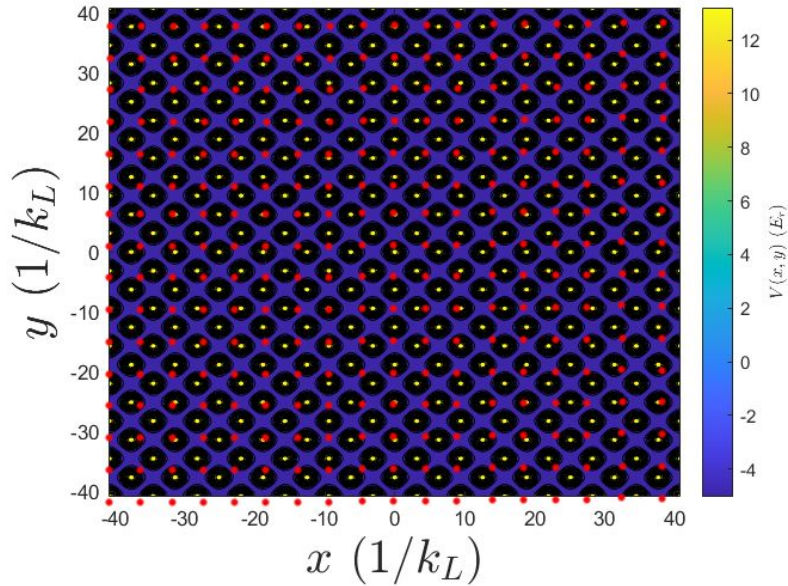
```
Nx: 2048 dtR: 0.01 ms F: 0.5  
dx: 0.08 dtF: 0.01 ms speed: 0.01x 16 ms  
a: 9.8832 tRamp: 80 ms tFree: 60 ms ramp
```



```
Nx: 1024 dtR: 0.01 Fx:  
dx: 0.08 ms 0.0070711  
ax: 9.8832 dtF: 0.01 Fy:  
ay: 7.8315 ms 0.0070711  
tRamp: 80 ms speed: 0.01x  
2 ms 40 ms  
ramp
```



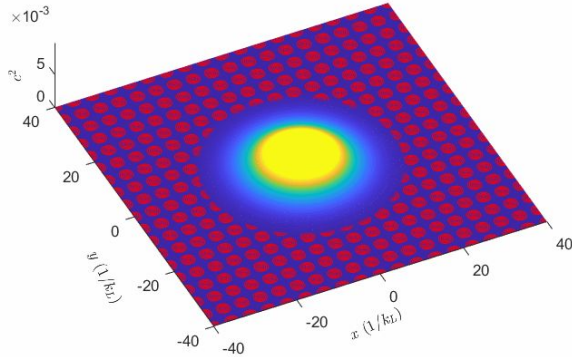
Importance of commensurate mesh



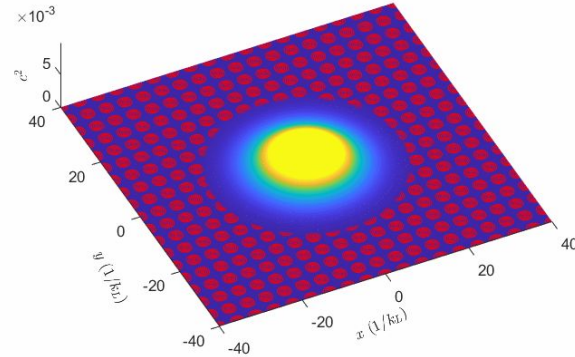
Results

Even with a commensurate mesh changing the angle of the force doesn't seem to introduce new or profound effects only changing the magnitude of the force seems to yield a significant change

Nx: 2048 dtR: 0.01 ms F: 0.1
dx: 0.08 dtF: 0.01 ms speed: 0.01x 26.7 ms
a: 9.8832 tRamp: 80 ms tFree: 60 ms ramp



Nx: 2048 dtR: 0.01 ms F: 0.001
dx: 0.08 dtF: 0.01 ms speed: 0.01x 26.7 ms
a: 9.8832 tRamp: 80 ms tFree: 60 ms ramp

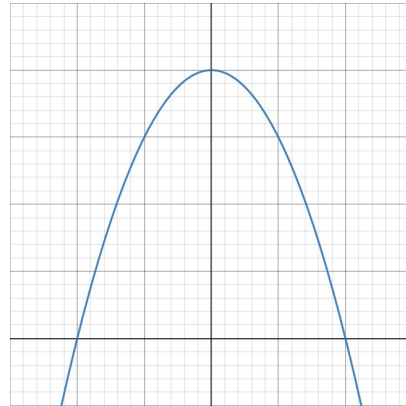
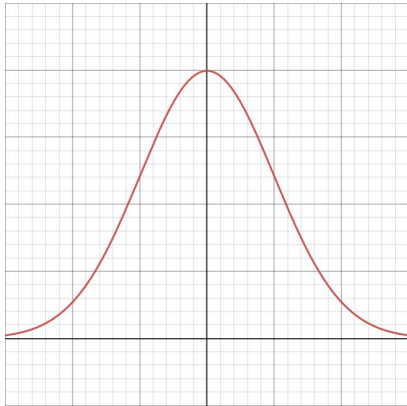




Next steps: Particle Interactions

The Gross Pitaevskii Equation (GPE): $i\frac{\partial}{\partial t}\Psi(x,t) = (-\nabla^2 + V(x,t) + g|\Psi(x,t)|^2)\Psi(x,t)$

Ground-state wave function is no longer a harmonic oscillator, but instead is a Thomas-Fermi distribution



Conclusions

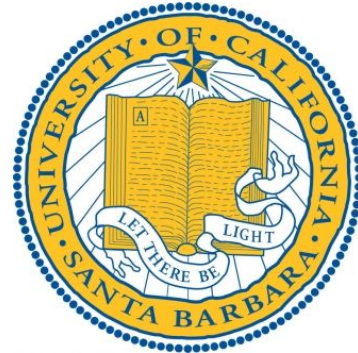
- ❖ Using small forces yields expected cloud behavior and shows little signs of unusual or chaotic nature
- ❖ Applying a force at varying angles may yet show significant results given the readjustment of the lattice
- ❖ Due to the irrationality of coefficients involved in the meshgrid our previous simulations need to be redone in case interesting dynamics were being overlooked

Future Work/ Next Steps

- ❖ With the GPE create framework to include particle interactions
- ❖ Run simulations using the reworked lattice to further explore the work already done

Acknowledgements

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