Visualizing Black p-Branes in String Theory

Abel Shiferaw, Georgia Tech

Mentors: Professor Don Marolf, Molly Kaplan, and Zhencheng Wang

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   B. Why Study Black Holes in String Theory?
2. Ramond-Ramond p-Branes in String Theory
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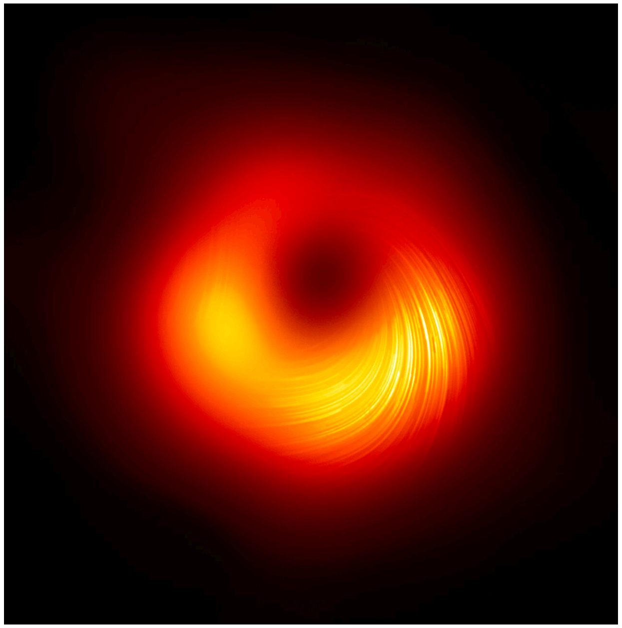
3. Story of an Infalling and Asymptotic Observer
   E. Asymptotic Flatness
   F. Gravitational Time Dilation (Redshift)
   G. Singularity



Black hole at the center of M87
(Event horizon telescope, 2017)

I: Intro to Black Holes

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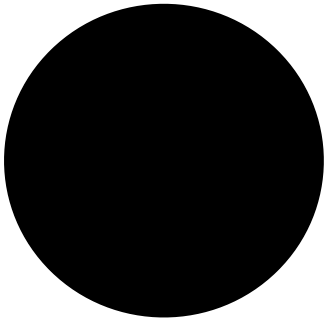

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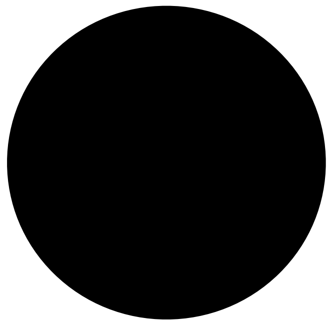

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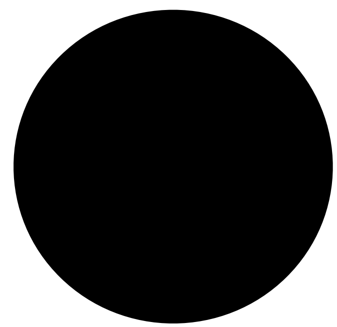

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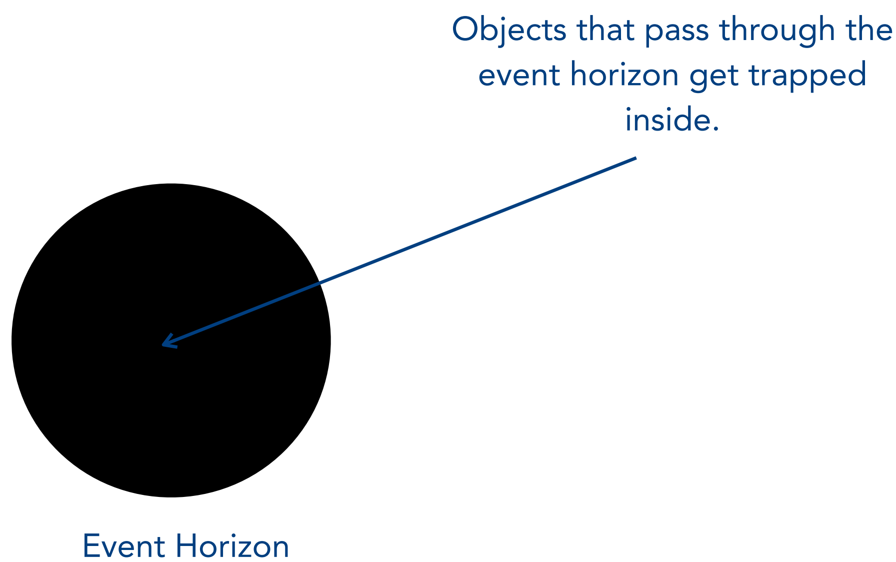

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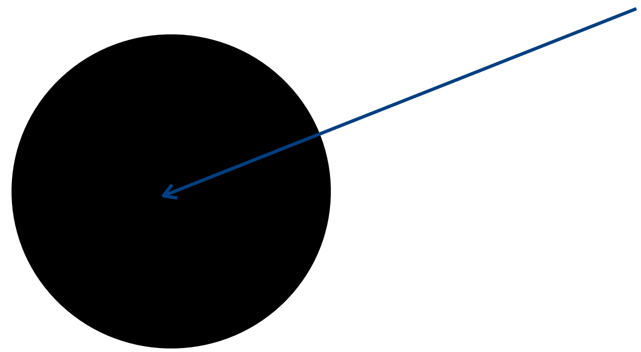

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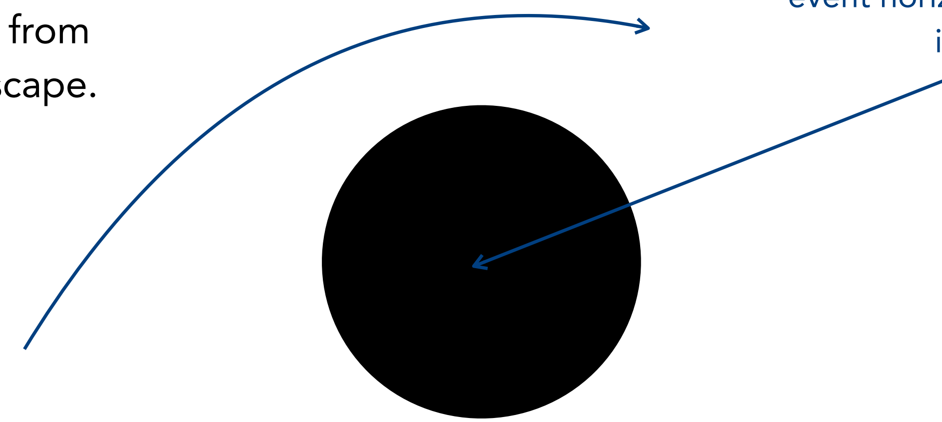

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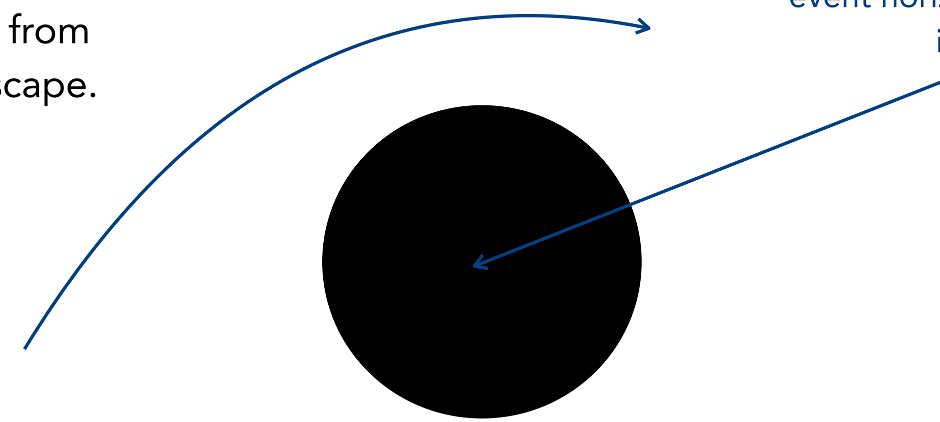

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Any object entering a black hole will **inevitably** arrive at the singularity, where the spacetime curvature becomes infinite, and all of our understanding of physics breaks down.

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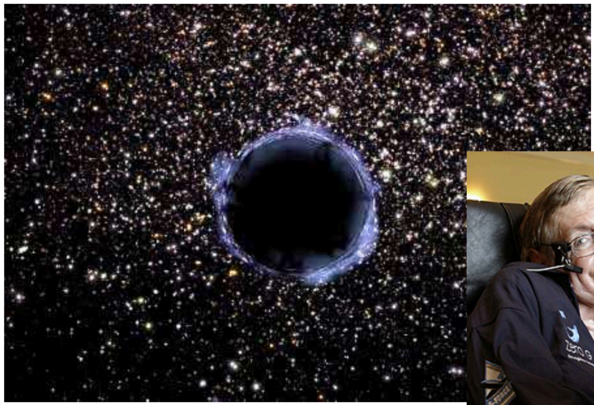

Steven Hawking (1942 - 2018)

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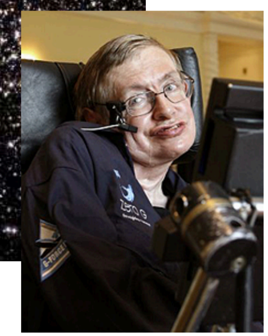
String theory, as a candidate theory of quantum gravity, has proven to be a useful tool for understanding this puzzle (**e.g., providing a microscopic description of black hole entropy via p-branes with Ramond-Ramond charge**).



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II. Black p-Branes in String Theory

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$$M/V_p = \frac{(8-p)\Omega_{8-p}}{2\kappa_{10}^2}r_H^{7-p}\left(1 + \frac{7-p}{8-p}\sinh^2\beta\right) \quad \text{and} \quad Q = \frac{(7-p)\Omega_{8-p}}{2\kappa_{10}^2}r_H^{7-p}\sinh\beta\cosh\beta.$$

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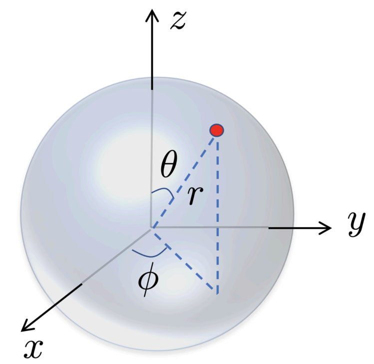

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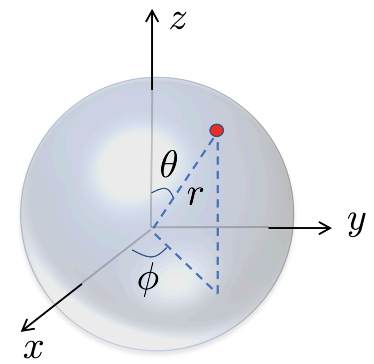

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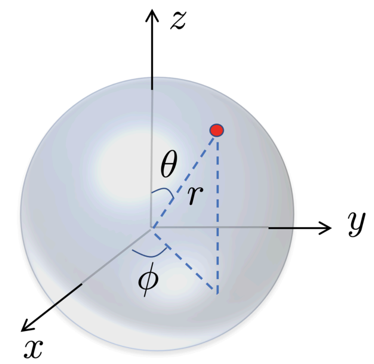

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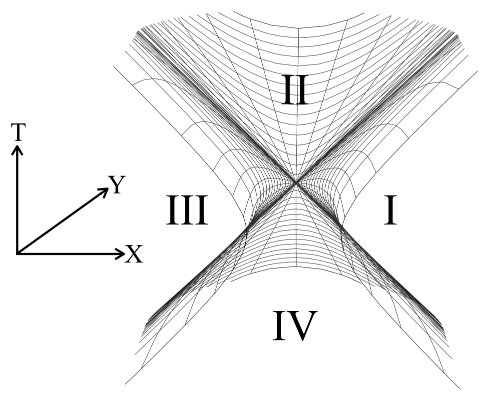

III: Story of an Infalling and Asymptotic Observer

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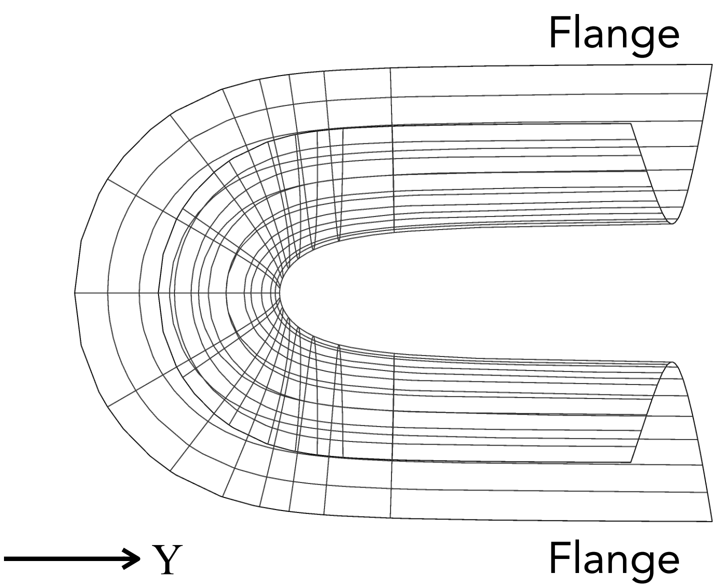

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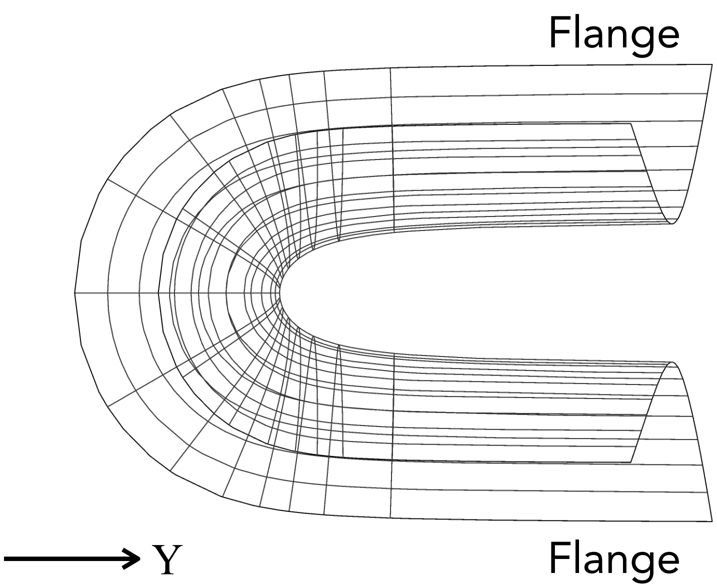

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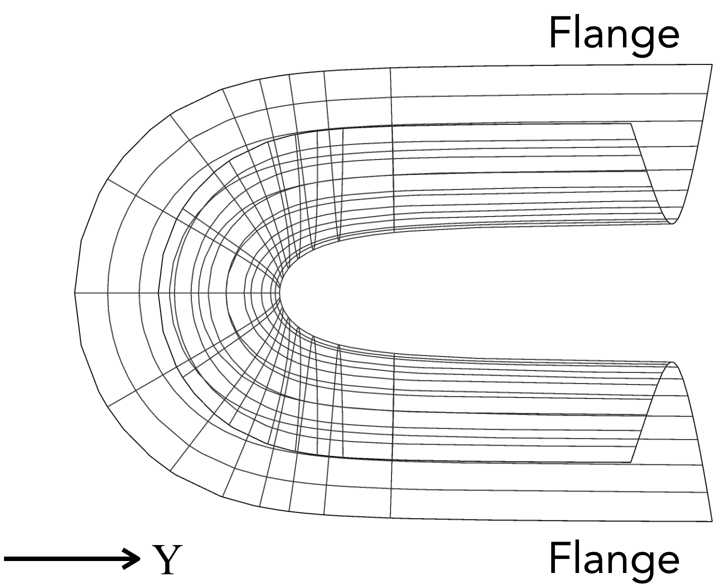

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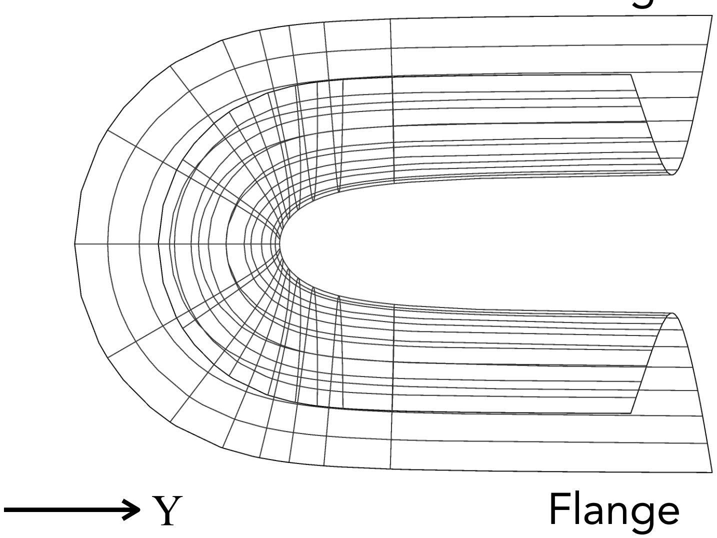

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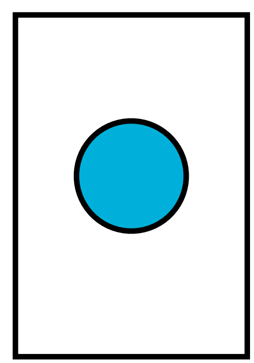

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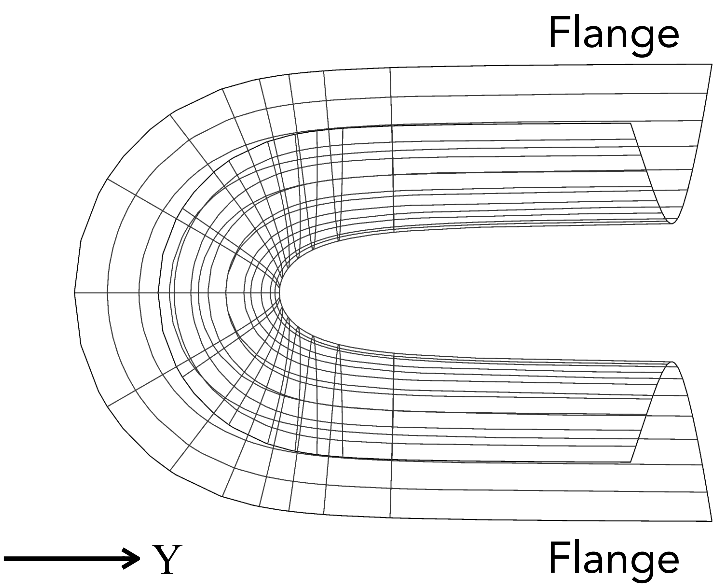

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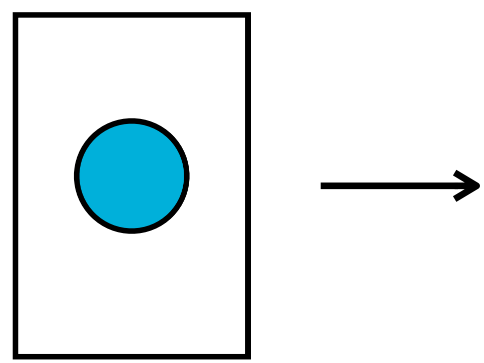

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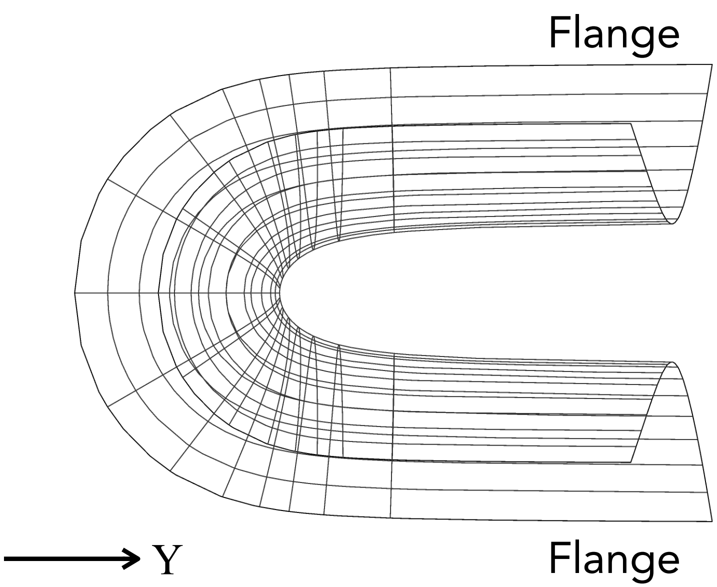

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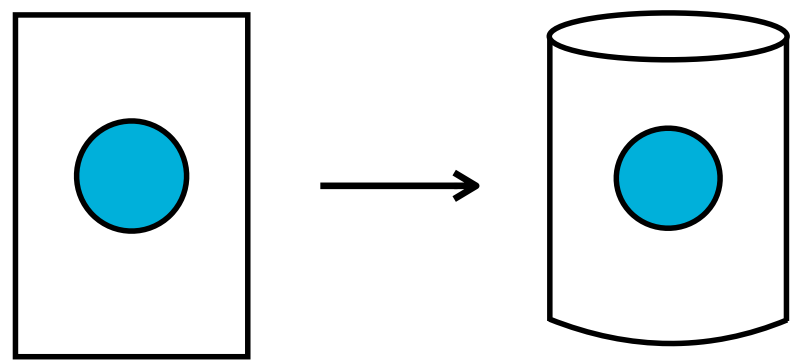

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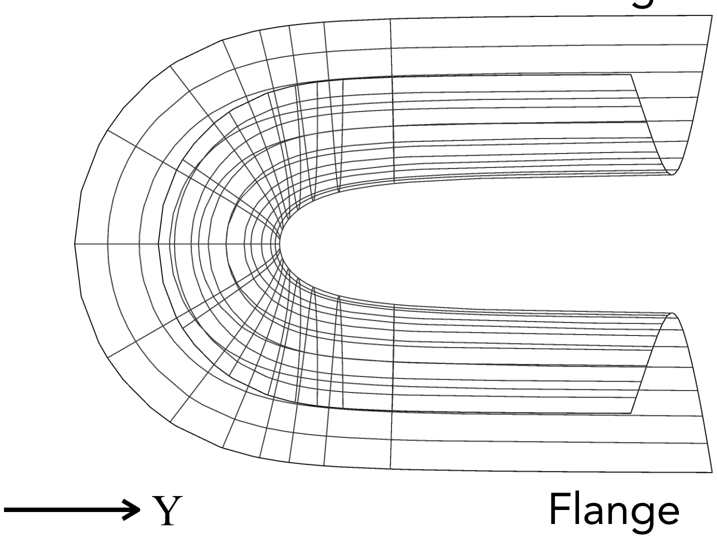

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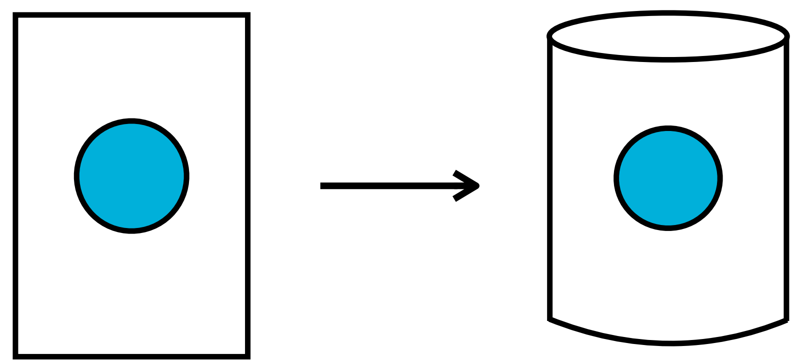

Deforming a surface in only one direction does not change the intrinsic geometry!

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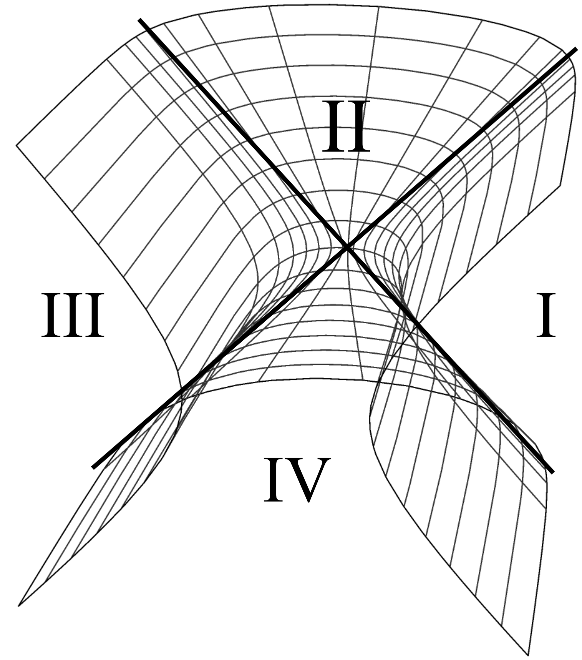

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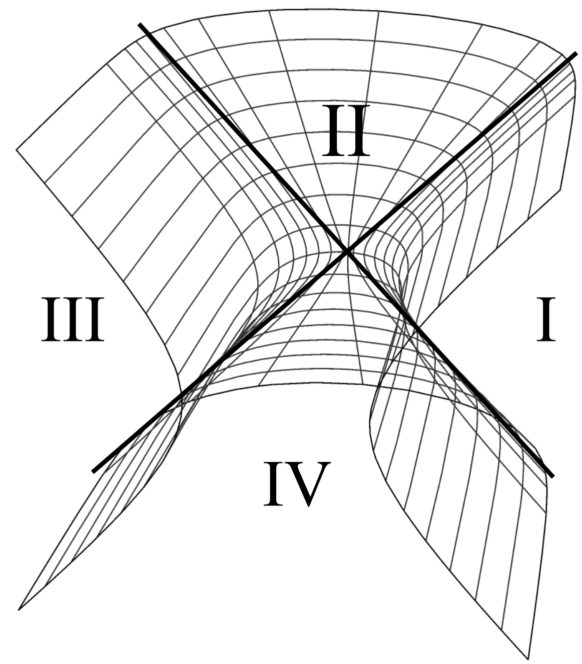

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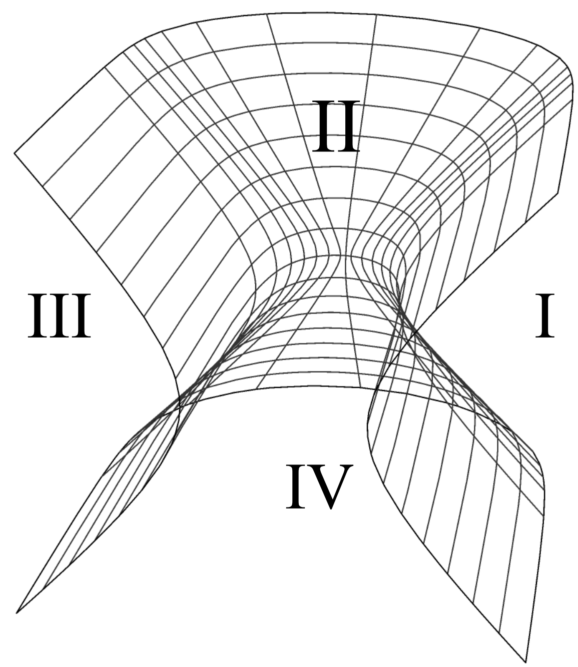

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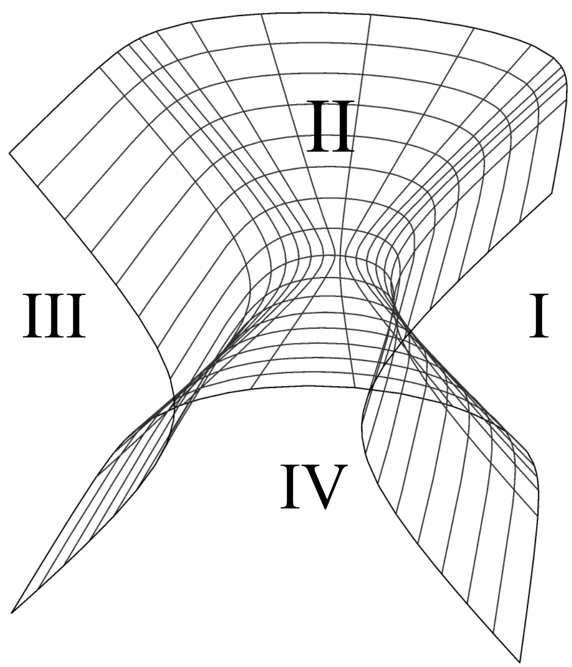

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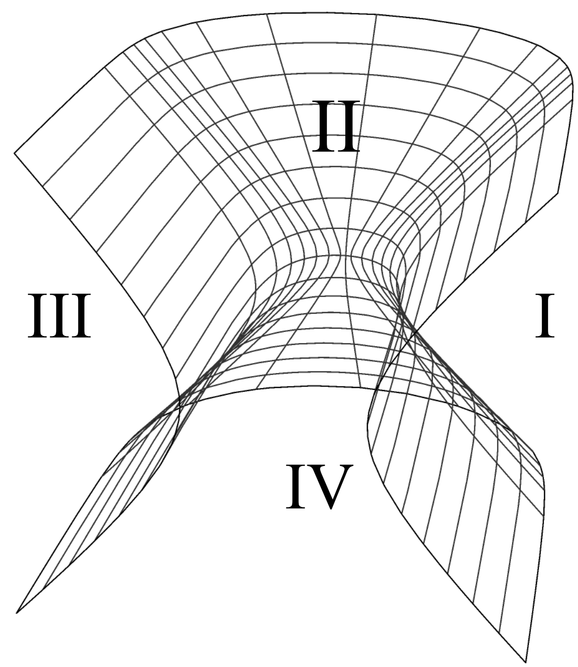

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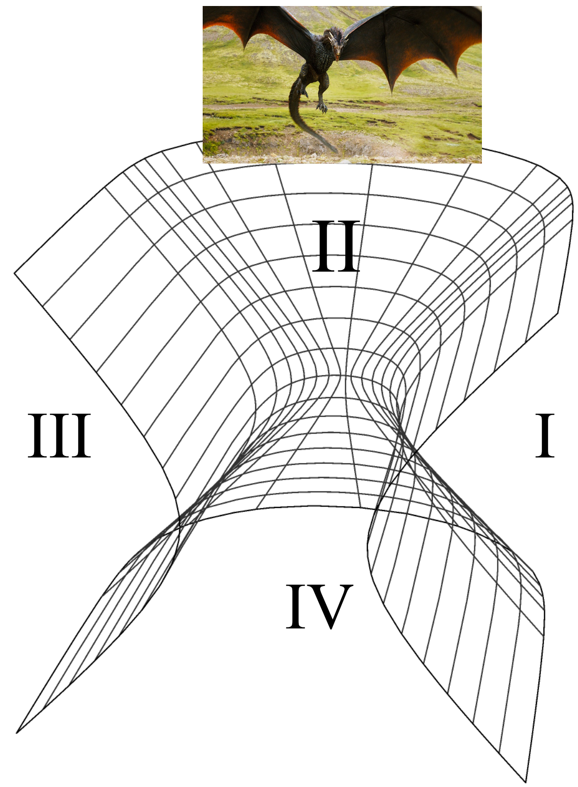

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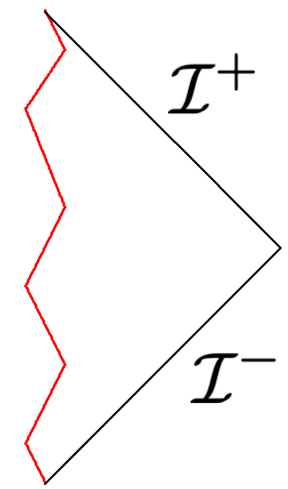

Conformal diagram for the extreme six-brane.

Credit: https://arxiv.org/abs/1107.1022

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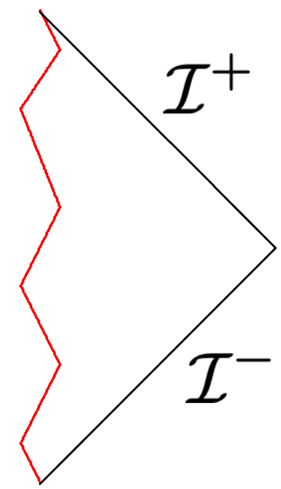
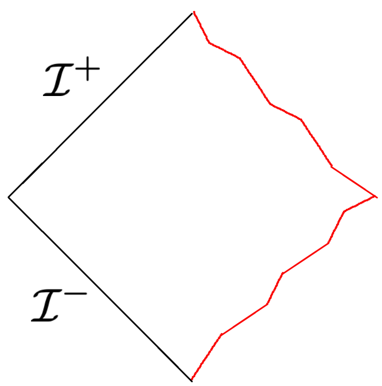


Conformal diagram for the extreme six-brane.



Conformal diagram for the extremal branes with $p < 6$.

Credit: https://arxiv.org/abs/1107.1022

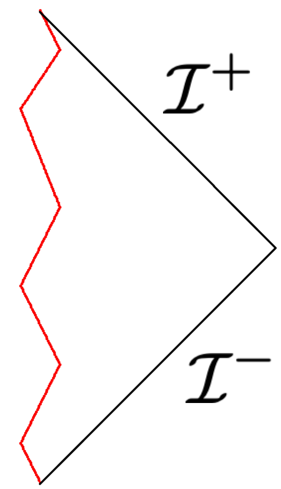
Future Work

What is the behavior of the embedding diagrams in the extremal limit ($\beta \to \infty$, $r_H \to 0$ with Q held fixed) as we vary p?

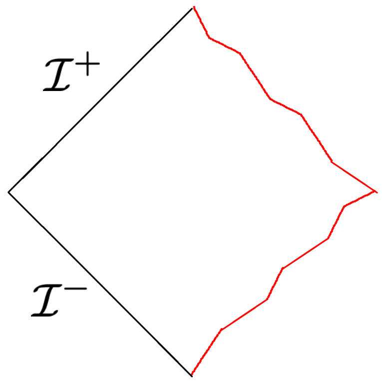
Analysis of causal structure shows that for $p = 6$, the extreme black brane contains a *timelike* singularity, whereas it has a *null* singularity for $p < 6$.

Do spacetime embedding diagrams know about this?

Credit: https://arxiv.org/abs/1107.1022



Conformal diagram for the extreme six-brane.



Conformal diagram for the extremal branes with $p < 6$.

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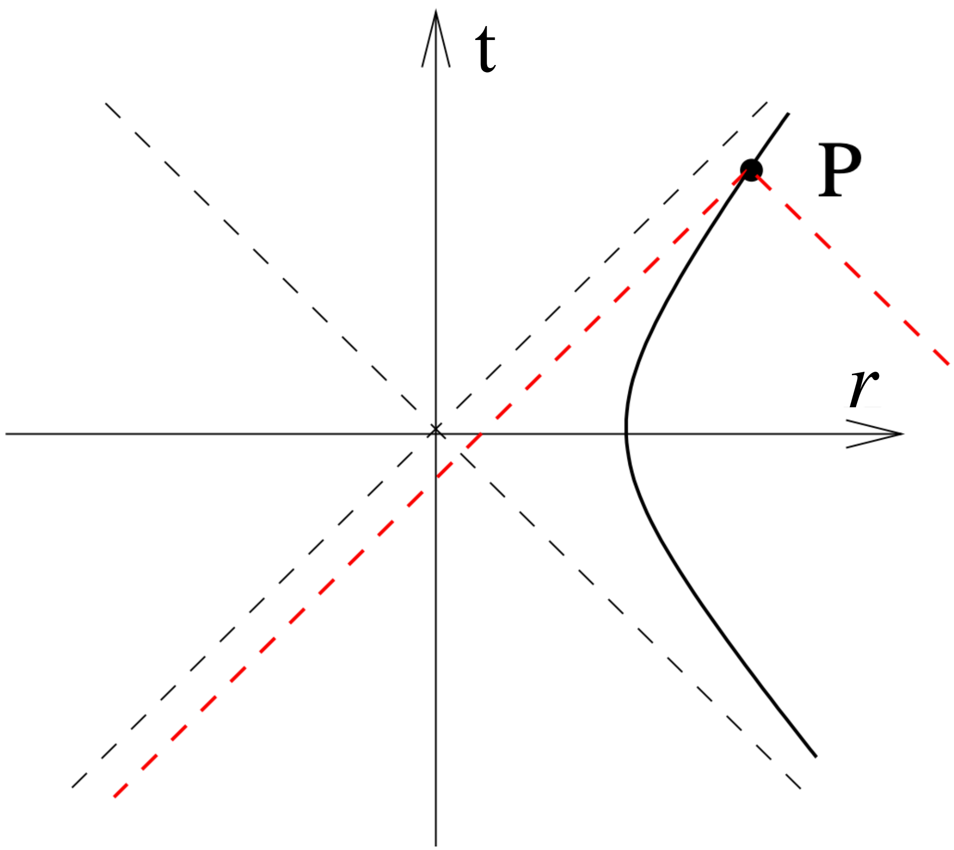
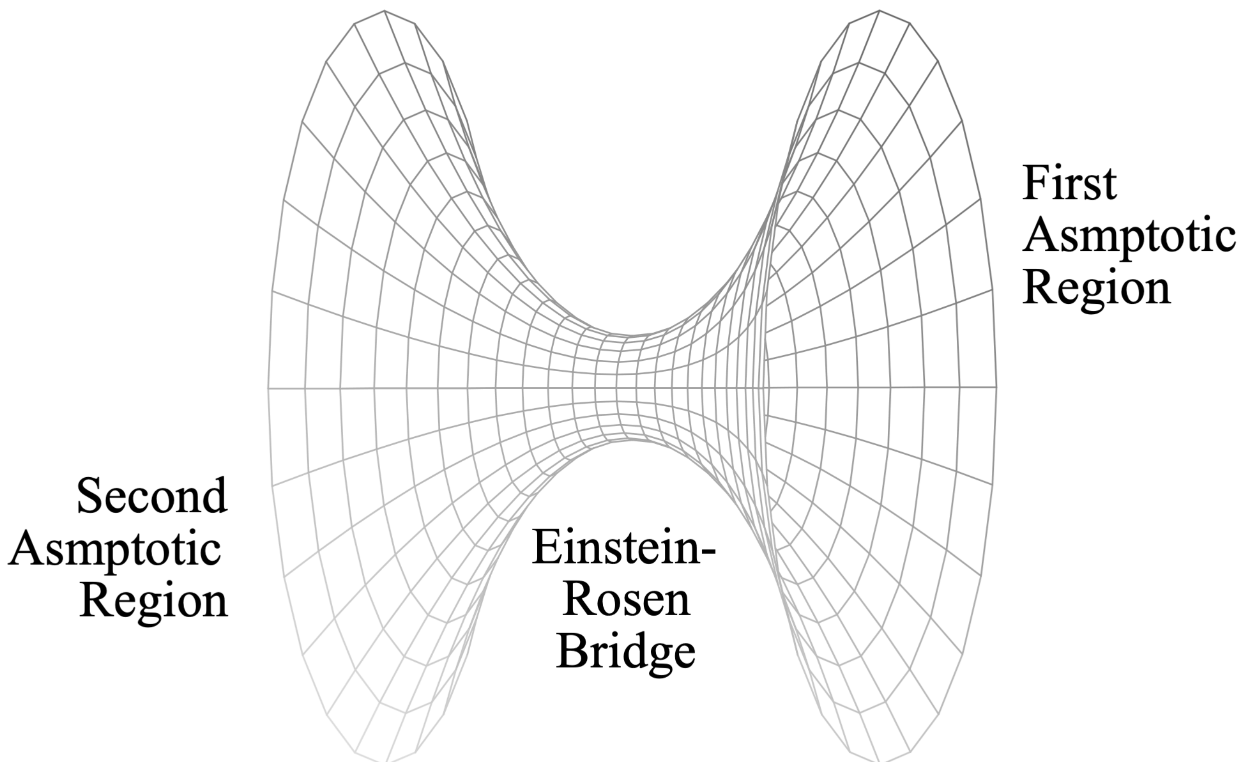
Thank you!

Extra Slides

Rindler Horizon

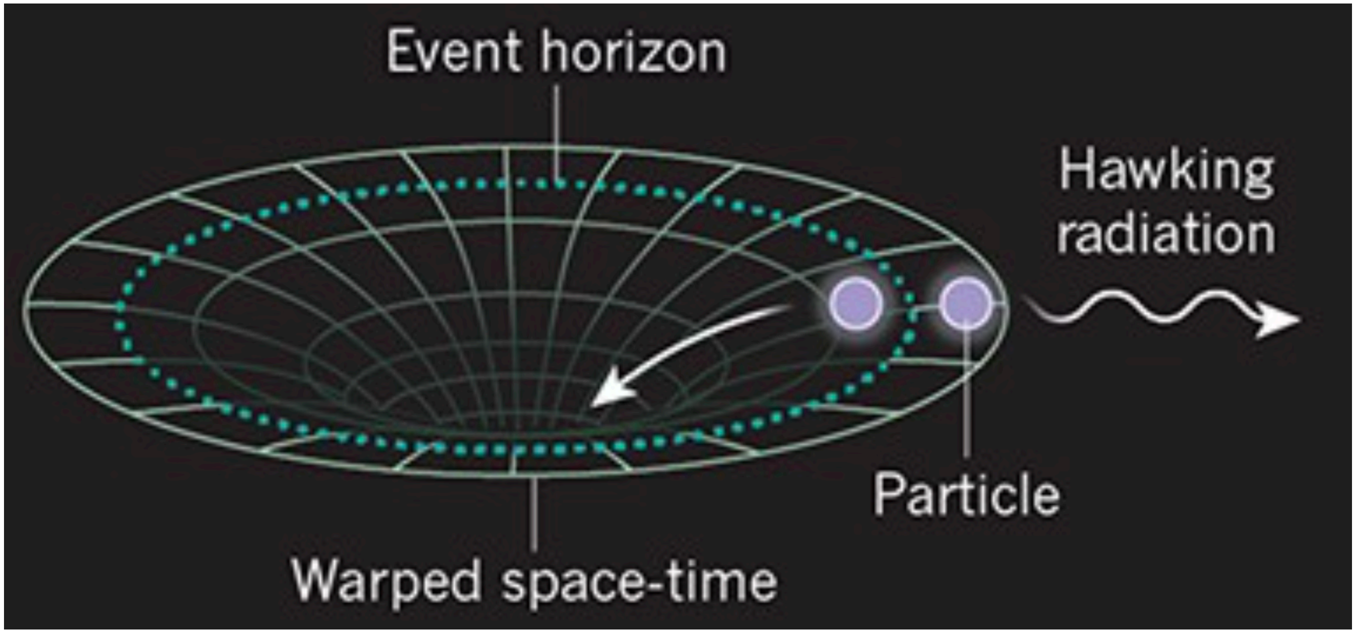


Wormhole



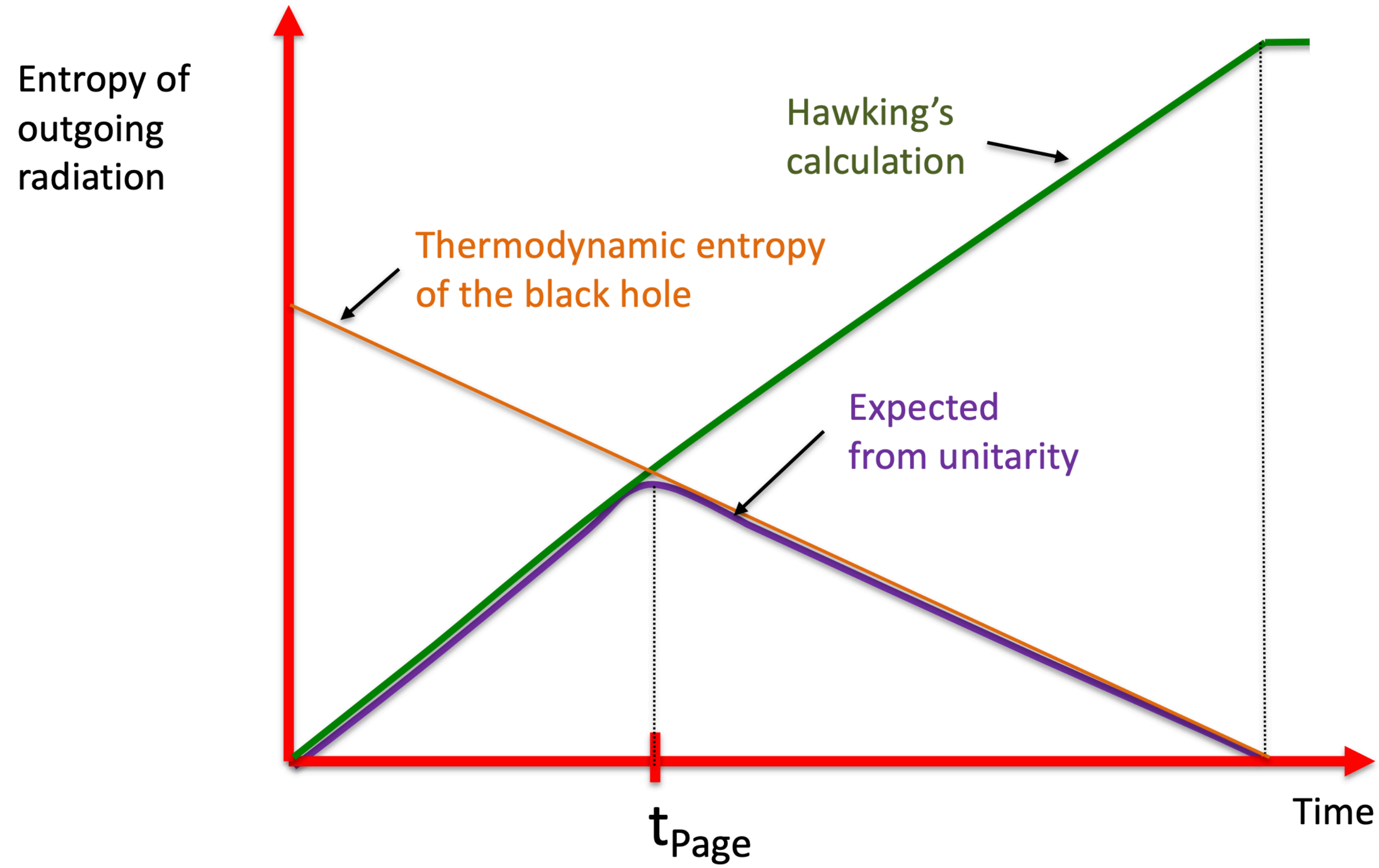
Credit: arXiv:gr-qc/9806123

Hawking Radiation



Credit: https://www.nature.com/articles/d41586-019-01592-x

Page Curve



Credit: https://arxiv.org/abs/2006.06872